THE UNIVERSITY OF ALBERTA

TILL FABRIC STUDIES IN THE EDMONTON AREA, ALBERTA, WITH SPECIAL EMPHASIS ON METHODOLOGY

by

JOHN RAMSDEN, B.A.Sc.

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF GEOLOGY

EDMONTON, ALBERTA

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ABSTRACT

As part of a continuing program of Quaternary stratigraphic studies in the Edmonton area, Alberta, till fabric determinations were made in an attempt to distinguish the till units. As the local stratigraphy became established, emphasis of the till fabric work was changed to concentrate on (1) examination of possible models for the statistical treatment of till fabrics, (2) investigation of the problems of determining the pebble fabric at an exposure and of interpreting the fabric pattern, (3) evaluation of till fabrics as an aid to local stratigraphic studies, and (4) establishment of local ice-movement directions.

Two mathematical models were examined to determine their value as bases for calculating descriptive statistics to represent till fabric data. A previously used model, that of a single axially symmetric spherical normal (Fisher) distribution, was examined and found unsuitable since most of the fabrics encountered were not unimodal. The testing of a model consisting of several Fisher distributions, one corresponding to each mode of the fabric, revealed that most of the fabric modes lacked axial symmetry.

The orientation of 550 elongate pebbles from a 300-foot exposure of a single till unit (six feet thick) revealed that: (1) The fabric varied both laterally and vertically, but when samples were taken together they showed a preferred trend parallel to groove molds at the base of the till. (2) Larger pebbles exhibited less scatter in their orientations than did smaller ones. (3) The orientation of the a-axis of a pebble was in part determined by the elongation of the pebble.

Pebble fabrics appear to be of little help in the recognition and

differentiation of Quaternary stratigraphic units in the Edmonton area.

Upper till fabrics are consistent with ice movement from the north-east as indicated by sole markings and surface features. Transverse fabrics were found at four out of 10 locations.

The direction of movement of the glacier that deposited the lower till cannot be reliably determined from the fabric evidence presented here. Interpretation of lower till fabrics is made difficult by the fact that deformation of this deposit occurred when the area was overridden by the glacier that laid down the upper till. Distinction between primary and wholly or partially altered lower till fabrics is a major problem in the study of lower till fabrics.

ACKNOWLEDGEMENTS

I am indebted to my supervisor Dr J A Westgate for his continuing advice, criticism and support during the course of this research. I wish to thank him also for his patience and encouragement while I pursued at some length my interest in the problems of statistical treatment of till fabric data, and the development of related computer programs.

Thanks are also due to Dr H A K Charlesworth of the Geology Department, University of Alberta, for helpful discussion and suggestions concerning the statistical treatment of the data and its implementation using the computer, and for bringing to my attention previous work in related fields.

Much of the data forming the basis of this study was obtained from gravel pits owned by Alberta Concrete Products Ltd., Twin Bridges Sand and Gravel Co. Ltd., J Pawluk and Son, and Mr G Kropp.

During the course of this research I held a Graduate Teaching Assistantship in the Department of Geology at the University of Alberta. Financial support was also received from the National Research Council of Canada.

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INTRODUCTION

Historical Review

Till Fabrics

On March 8, 1856 Henry Youle Hind presented a paper to the Canadian Institute entitled "On the blue clay of Toronto" in which he described the orientation of the stones and suggested a glacial origin for the deposit rather than deposition from floating ice or by the sea (Elson, 1966). Miller (1884) in describing "pavement boulders" in the till near Edinburgh, Scotland, stated (p.167): "The longer axis of the stone is often directed in the line of glaciation, and the pointed end is frequently, but not always, toward the ice." He also suggested that the orientation of the stones might reflect their original orientation in the ice. Bell (1888) observed a tendency for boulders on Swiss glaciers to assume a longitudinal position. "Upham (1891) described the characteristic position of 'oblong' stones in sub-glacial till as having their long axes parallel to contiguous striae. Flat stones were said to lie parallel with the surface of deposition." (Holmes, 1941, p.1302).

The first systematic investigation of till fabric appears to have been carried out by Richter in the early 1930's. According to Holmes (1941, p.1302) Richter's conclusions confirmed the earlier observations of Miller and Bell and were supported by quantitative statistical data. Richter was apparently the first to note a tendency to an orientation transverse to glacier flow.

Holmes (1941) investigated the relationship between the orientations of pebbles in a till and their physical properties. Using a total of 1180 pebbles from till of central New York he attempted to demonstrate that groups of pebbles having different combinations of physical properties (shape, rounding, axial ratios, size) had significantly different preferred orientations. His work drew attention to many important questions, and led him to suggest the "plastering-on" process of till deposition.

In an attempt to answer some of these questions, Harrison (1957) studied the orientation of disk- and blade-shaped pebbles having b:c ratios greater than two. He felt that "particles of this shape are better indicators than any others of dynamic forces responsible for their orientation." (Harrison, 1957, p.276). His work showed that disk- and blade-shaped particles tend to lie in a near-horizontal plane and to be imbricated upstream to ice movement. Harrison favoured the theory that the orientations of the pebbles in a till reflect the orientations that they achieved in the ice.

Glen, Donner and West (1957) showed that the observed orientations of till pebbles could be entirely the result of flow in moving ice. Collisions between particles and the mode of deposition could change the distribution, however.

More recently, attention has been given to till fabric variability, the reliability of fabric determinations, and problems of sampling. For example, Kauranne (1960), Andrews and Smith (1966), Andrews and King (1968), Johansson (1968) and Harris (1969) have investigated till fabric variability. These studies have shown that till fabrics may be highly variable, in both preferred orientation and degree of preferred orienta-

tion. Thus any till exposure must be regarded as a unique problem in sampling.

The problem of determining sediment origin from pebble fabric patterns has apparently received little attention. Lindsay (1968), in relation to simulation of mudflow pebble fabrics by computer, briefly considered the problem of differentiating between till pebble fabrics and mudflow pebble fabrics, but the work was inconclusive. Recently, with more promising results, he compared the pebble fabrics of two diamicton deposits with known till fabrics to test the theory that the deposits were tillites (Lindsay, et al., 1970).

Previous work on the Quaternary sediments of the Edmonton area (fig. 1) has been summarized by Westgate (1969). Although the basic stratigraphy has been established, little work has been done on the till fabrics of the area. Till fabric data obtained by other workers in recent years has been incorporated in this report.

Statistical Treatment of Geological Orientation Data

Much work has been done on the statistical treatment of orientations in two dimensions. One important method involving vector summation was apparently first used by Reiche (1938) and was applied to the orientations of pebbles in sediments, including till, by Krumbein (1939). A second powerful method developed by Tukey (1954) using a combination of the chi-square technique and vector summation was applied to the orientations of till pebbles by Harrison (1957). A simple chi-square test was used on till fabric data by Kauranne (1960) and forms the basis of a method described by Harris (1969) for determining minimum satisfactory sample sizes.

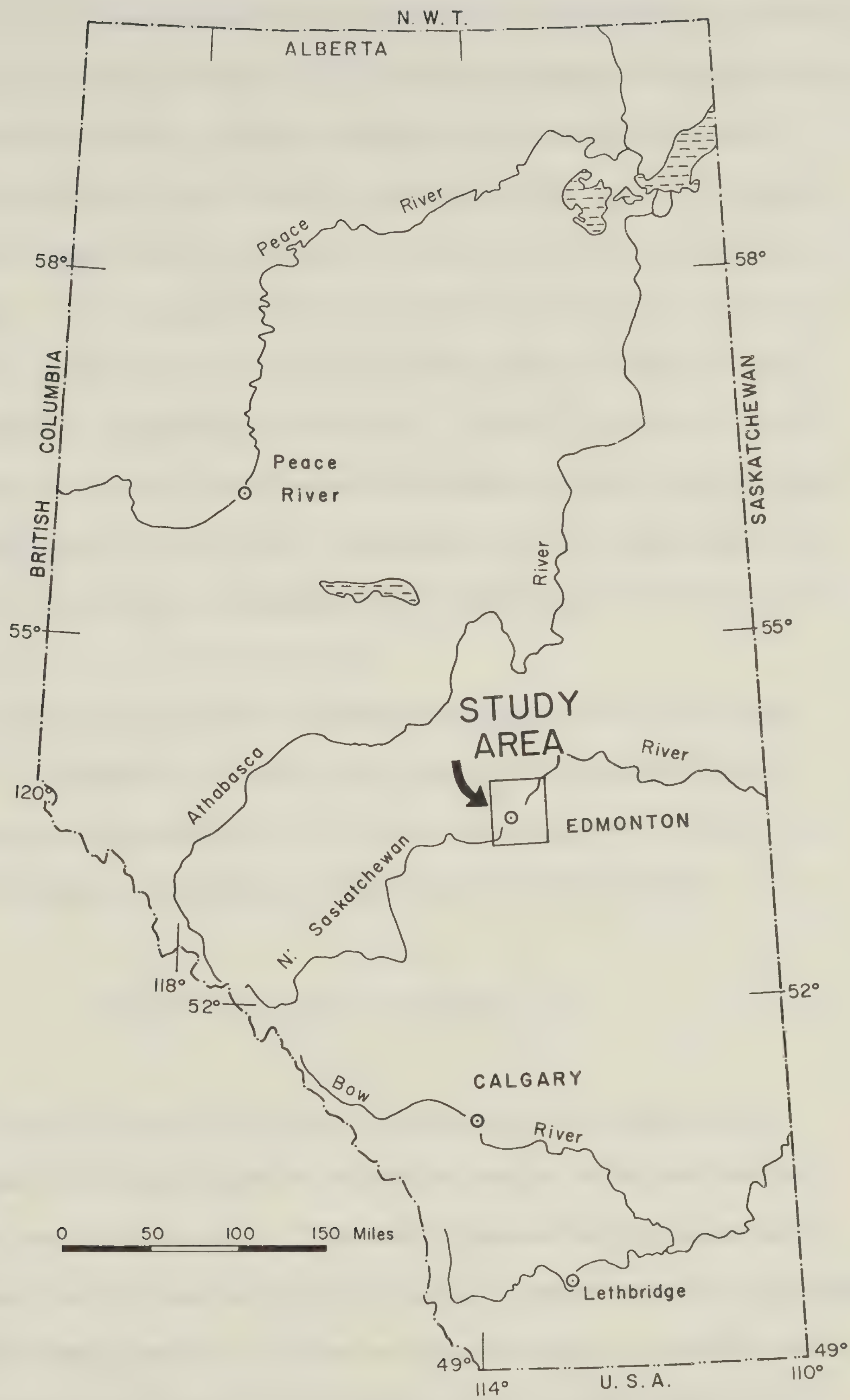


Figure 1 Location of study area.

The present study deals with methods of treating the orientations of pebble axes in three dimensions. For the treatment of directions in three-dimensional space, spherical probability distributions were constructed by Arnold (1941) but were not published until included by Pincus in his comprehensive review of methods of analyzing orientation data in 1953. In the same year Fisher (1953) developed the theory of the spherical normal model as applied to the axially symmetric case for the treatment of geomagnetic vectors. Further statistical procedures based on Fisher's work were developed by Watson (1956a, 1956b, 1960), Watson and Williams (1956), and Watson and Irving (1957). A somewhat different model was suggested by Scheidegger (1965) designed for the treatment of non-directed axes.

The procedure described by Watson and Irving (1957) for the treatment of paleomagnetic vectors was adopted by Steinmetz (1962) for representing measurements of direction and angle of dip of cross-beds, and by Andrews and Shimizu (1966) for till fabric data.

Objectives of Present Study

The initial objective of this study was to map exposures of glacial deposits in the Edmonton area so as to get a better understanding of the stratigraphy. Till fabric determinations were made in an attempt to distinguish the till units. As the local stratigraphy became established, emphasis of the work was changed to concentrate on (1) evaluating the reliability of the till fabric measurements, (2) assessing the usefulness of till fabrics in the local stratigraphic studies, and (3) establishing local ice movement directions.

Later a fourth major objective was added: the application and evaluation of Andrews and Shimizu's (1966) method of calculating descriptive statistics to represent till fabric samples and serve as a basis for comparison of samples.

The Approach

Since the principal objective initially was to map as many new sections as possible, only one fabric measurement was made on each till unit at each site, except in the case of certain important exposures where it was considered worthwhile obtaining more detailed fabric information.

The reliability of the till fabric determinations was investigated by taking duplicate samples from certain sites to test the adequacy of 50-pebble samples, and by measuring both lateral and vertical sequences of fabrics to test the within-site variability and the value of single samples. At one location, the orientations and dimensions of about 200 pebbles were measured to see if any effects of pebble shape or size could be detected in the fabric patterns.

Based on a FORTRAN IV program developed at the University of Alberta for producing stereographic projections of point density, a modified program was developed that would take account of measurement errors and calculate the probable density of true axes rather than the actual density of recorded axes.

Andrews and Shimizu's (1966) adaptation of Watson and Irving's (1957) procedure for calculating descriptive statistics for three-dimensional orientations was employed. Statistics calculated by the

method were compared with point density diagrams of the data in order to evaluate the success of the method. An alternative method devised by the writer based on a different assumption about the distribution of pebble orientations was also used and evaluated.

PROCEDURES OF DATA COLLECTION AND GRAPHICAL REPRESENTATION

Data Collection

Definitions

The dimensions and shape of a pebble are described by the lengths of three imaginary mutually perpendicular "axes". The c-axis is perpendicular to the maximum projection plane (Krumbein, 1939, p.677) of the pebble. The a-axis lies in this plane, and is parallel to the longest dimension of the pebble in this plane. The b-axis is perpendicular to both the a- and c-axes. The a-, b-, and c-axes are also referred to as the long, intermediate, and short axes respectively. The planes perpendicular to these axes are referred to as the bc-plane, the ac-plane, and the ab-plane.

The lengths of the three axes are indicated by the lower case letters a, b, and c. Pebble shape is often described by the ratios of these lengths, for example, the length of the a-axis divided by the length of the b-axis is called the a:b ratio or simply a/b. The a:b ratio is here called the "elongation" of a pebble, and the b:c ratio the "flatness".

The definitions of the a-, b-, and c-axes used here are the same as those described by Krumbein (1939). However, it should be noted that Krumbein's statement that "the a-axis is normal to the minimum projection plane" (Krumbein, 1939, p.678) is not necessarily true.

Selection and Measurement of Pebbles

At most sites only the orientation of the a-axis of each pebble was measured. At location F, the lengths a, b, and c were recorded for some of the pebbles, and for some of these the attitude of the ab-plane was recorded in addition to the orientation of the a-axis.

Particles smaller than about one quarter inch were not measured due to the difficulty of obtaining accurate measurements. Pebbles smaller than about one inch were measured only when their use was necessitated by the scarcity of larger pebbles. Pebbles with an a:b ratio of less than about 1.2 were not measured, except at location F as described in Chapter 6.

Most samples consisted of measurements of 50 pebbles. The volume of till from which a sample was taken varied from about one cubic foot to about six cubic feet, depending on the abundance of pebbles and the condition of the till.

The number of samples taken from one till or till-like unit at a locality varied, the minimum being one and the maximum 11. When two or more samples were taken from a unit at a single location, they were located so as to yield the greatest amount of information about both vertical and lateral fabric variability within the limits of the exposure.

Field Technique and Sources of Error

Care was taken to select sample sites in undisturbed till, avoiding slumped material. After a sample site had been selected,

the existing surface of the outcrop was removed to a depth of about six inches, or as much as was necessary to expose fresh, undisturbed, workable material.

The procedure of exposing and measuring pebbles was as follows. The till was removed using a small pointed trowel until a pebble was encountered. The pebble was held firmly in position while the surrounding matrix was carefully removed until about half of the pebble was exposed. The pebble was then removed, while care was taken to preserve its mold in the remaining matrix. At this point the pebble was discarded if it had too small an elongation to be usable. If the pebble was usable, the position of its long axis was estimated, but not marked on the pebble. The pebble was then carefully replaced in its mold and the trend and plunge of its long axis were measured to the nearest degree using a Brunton compass. In some early samples the trend and plunge were measured to the nearest five degrees. The sources of error in the measurements thus obtained are thought to be as follows.

1. Error involved in replacing pebble in its original position. This error was probably less than two degrees in most cases.
2. Error involved in estimating the position of the long axis after replacement of the pebble. This error was highly variable, depending on the shape of the pebble, how much of the pebble was exposed, which portion of the pebble was exposed, and the attitude of the pebble in relation to the outcrop surface (that is the angle from which the pebble had to be viewed by the observer). The maximum probable value of this error ranged up to about five degrees.

3. Error in aligning Brunton compass with estimated position of long axis. This error was probably less than two degrees in all cases.
4. Error in reading scale: one half degree.

Thus the discrepancy between the recorded orientation of an axis and the true orientation of the axis was considerably less than 10 degrees in all cases.

Graphical Representation: Contoured Point Density Diagram

Introduction

Initially, the writer obtained equal-area projections of point density using a FORTRAN IV program developed at the University of Alberta (Cruden, 1966, p.84). This program determines point density by counting axes or the poles to planes directly on the reference hemisphere. The axis of a circular cone whose apex is at the centre of the hemisphere is placed successively through each of 333 counting locations, and the number of observations falling within the cone at each location is recorded as a percentage of the total. The solid angle contained within the cone can be set to any desired fraction of the total solid angle of the hemisphere. This cone defines a circle on the surface of the hemisphere whose area is the same fraction of the total surface area of the hemisphere. The program uses the computer's line printer to produce a 10-inch radius equal-area projection of the counting locations, each location being represented by

the appropriate point density value. To give a simple line-printer format, the counting locations were chosen so that their projections form a rectangular grid with a spacing of one tenth the radius of the reference hemisphere. Sixteen locations near the periphery of the projection were added to make a total of 333.

Suppose that a sample of N axes is taken from a population of axes, and that the program is used to prepare a density diagram using a P per cent counting cone (or circle). The contoured diagram then represents a "density surface", the elevation of which at any point shows the percentage of axes in the sample falling within a P per cent counting circle centered at that point, and gives the probability of an axis selected randomly from the sample falling within a P per cent counting circle centered at that point. It is taken as an estimate of the probability that an axis selected randomly from the population will fall within a P per cent counting circle centered at that point.

This method of determining point densities is a great improvement over previous methods of counting points on a projection. Previously, accurate contouring on an equal area projection required an elliptical counter whose eccentricity increased progressively with distance from the centre of the projection, so that use of a circular counter introduced inaccuracy that was greatest at the periphery and decreased to zero at the centre. Since the present method counts the points directly on the reference hemisphere using a circular cone, all such inaccuracy is eliminated. Use of a computer also eliminates plotting and counting errors and allows diagrams to be prepared in a small fraction of the time required to prepare them manually.

Modified Procedure

In the program just described, a counter is incremented by one for each observation that falls within the counting circle, regardless of where in the counting circle it falls. Thus an observation falling just inside the counting circle, for example at point A in Figure 2, is counted as one, while an observation falling just outside the counting circle, for example at point B in Figure 2 is not counted at all.

With relatively small samples, this results in erratic contours. Also, in interpreting the diagram in terms of the distribution of axes in the population, it is assumed that the axes in the sample are identical to the axes that they represent in the population. In other words, errors of measurement are ignored, as are inherent variations in the attitude of the feature being measured. For example, when the attitude of a joint that is not perfectly planar is represented by a single measurement, that measurement is assumed to indicate the mean orientation of the joint: both measurement errors and the variation in attitude or "roughness" of the joint are ignored. It was felt worthwhile to develop a modification of the program that would take into account the distribution of errors in the measurements, thereby smoothing the contours and producing a diagram that is more likely to approximate the actual distribution of axis density in the population.

Method Used in the Program

Following a basic idea originally used by Dr. D. M. Cruden (Cruden, 1966), the writer developed a program that would take measurement

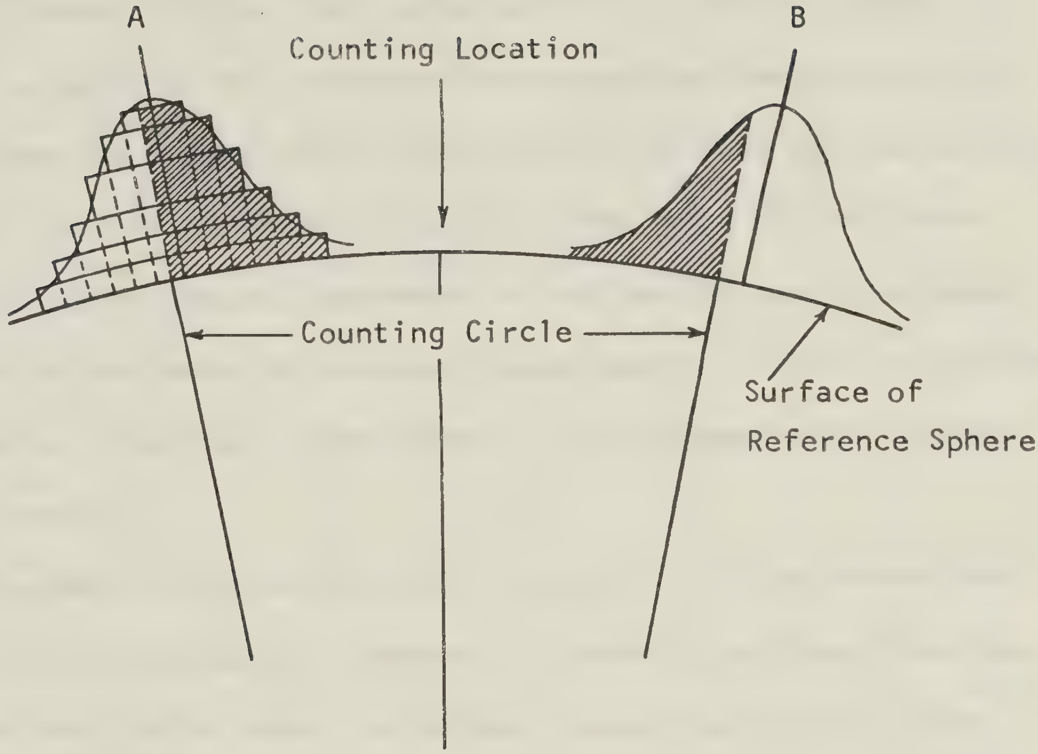


Figure 2. Section through centre of reference sphere, two sample axes A and B, and one counting axis, illustrating the difference between the methods of incrementing the counter in the modified and unmodified programs for preparing point density diagrams.

errors into account. The problem was to design a counting procedure that, rather than counting 1 for each observed axis that fell within the counting circle, would increment the counter by a quantity Pr, where Pr is the probability that the <u>true axis</u> represented by the observation lies within the counting circle. Errors in the measurement of an axis were assumed to conform to a spherical normal or Fisher distribution (Fisher, 1953). Each observation was then no longer thought of as a point on the surface of the reference hemisphere, but rather as a spherical normal probability density surface centred at the observed point, as illustrated in Figure 2. The shape of the Fisher distribution is described by a parameter K, called the precision parameter. K must be estimated by repeated measurements of selected pebbles. After a value for K has been established, it is assumed to be constant for all measurements.

If the probability of occurrence of a point in a two-dimensional sample space is represented by a probability density surface, then the probability of that point occurring within a specified region of the sample space is given by the volume beneath the probability surface over the specified region. For the modified point density program, the two-dimensional sample space is the surface of the reference hemisphere; the point is the true axis represented by an observation, and its probable position is described by a spherical normal probability density surface centred at the observed position; the specified region of the sample space is the counting circle. Hence the probability of the point (true axis) falling within the counting circle is given by the volume beneath that part of the probability surface that lies within the counting circle.

To facilitate computation of this volume, the program approximates the probability density function used to represent each true axis by a series of concentric discs arranged in a vertical stack, with the radii of the discs decreasing from bottom to top (fig. 2). Progressively larger discs are added to the bottom of the stack until the approximation contains more than 99.9 per cent of the volume beneath the probability surface, that is, more than 99.9 per cent of the probability.

For a given observation and a given position of the counting circle the required volume is determined by calculating how much of the area of each disc is contained within the counting circle and multiplying this area by the height of the disc, then adding the results for all the discs that overlap the counting circle. The result is a number between 0 and 1, representing the fraction of the total volume of the distribution that overlaps the counting circle. For a given position of the counting circle, this fraction is computed for each observation, and the fractions for all observations are added. The result is expressed as a percentage of the total number of observations, and recorded as the density value for that position of the counting circle.

Statistical Significance of Modified Procedure

It will now be shown that the counting procedure just described determines, for a specified position of the counting circle, the probability that the true axis represented by an observation selected randomly from a sample of N observations lies within the counting circle.

The selection from N observations of an observation whose true axis lies within the counting circle can happen in N different ways: (1) selection of the first observation, and its true axis lying within the counting circle, (2) selection of the second observation, and its true axis lying with the counting circle, and so on up to: (N) selection of the Nth observation, and its true axis lying within the counting circle. The occurrence of any one of these N events itself consists of the simultaneous occurrence of two events: (1) the selection of the particular observation, and (2) its true axis lying within the counting circle. The probability of the simultaneous occurrence of two events is given by the product of the probabilities of the individual events occurring alone. Hence the probability of obtaining a true axis within the counting circle in any one of the above N ways is $(1/N) \times Pr_i$, where $1/N$ is the probability that the ith observation will be selected, and Pr_i is the probability that its true axis lies within the counting circle. Furthermore, the N different ways of obtaining an axis within the counting circle are mutually exclusive, and the probability of the occurrence of at least one of a number of mutually exclusive events is given by the sum of the probabilities of the individual events. Hence the probability of obtaining an axis within the counting circle in at least one of the N mutually exclusive ways is $\sum_{i=1}^{N} \frac{1}{N} \times Pr_i$. Since N is constant, this is the same as $\frac{1}{N}\sum_{i=1}^{N} Pr_i$. But $\sum_{i=1}^{N} Pr_i$ is the quantity determined for each counting circle by the counting procedure described above, and hence when this is divided by N the result is the probability that an observation selected randomly from the N observations represents a true axis lying within the counting circle.

Determination of K

The meaning of K in terms of the tightness of grouping of the population it describes is illustrated by the following table. Each value of K is given with its corresponding value of ϕ_{95}, the significance of which is as follows. If an axis selected randomly from the population makes an angle ϕ with the central direction of the distribution, then ϕ_{95} is an angle such that $Pr(\phi < \phi_{95})=0.95$.

K	ϕ_{95} (degrees)
10	45.6
20	31.8
50	19.9
100	14.1
200	9.9
500	6.3
1000	4.5

So that the program can approximate the probability distribution representing the true axis of a pebble, the degree of scattering of repeated measurements, must be specified by means of the precision parameter K. Two sets of repeated measurements on single pebbles were made by the writer to test the assumption of a spherical normal distribution and to estimate K. These sets of measurements are labeled 991 and 992, and consist of 23 and 20 measurements respectively. For set 991, k, the best estimate of K, is 695. For set 992, k is 119. A value of K=500 was more or less arbitrarily chosen and used to prepare modified point density diagrams. The value of 6.3 degrees for ϕ_{95} implied by this value of K appears to be reasonable in view of the probable magnitude of errors discussed above.

Validity of Assumptions

It is thought likely that K varies from pebble to pebble, depending chiefly on the shape of the pebble. Other factors mentioned in the section on errors in the measurements may also have an effect. The fit to Fisher distributions with the estimated parameters was tested using the modified mean vector program described in Chapter 3 and given in Appendix D. Both sets 991 and 992 contained too few observations to allow a chi-square test of the distribution of the angles between the observations and the estimated mean direction. A chi-square value was calculated for each axial distribution about the mean, however. The chi-square value was 5.00 with one degree of freedom for set 991, indicating that there is a probability of less than five per cent that this set has a Fisher distribution. For set 992, the chi-square value was 13.60 with one degree of freedom, indicating a probability of considerably less than one half of one per cent that this set has a Fisher distribution. Hence the assumption that the measurement errors have a Fisher distribution is at best an approximation.

It is evident from the foregoing discussion that the two major assumptions involved in the modified procedure are both invalid when the technique is used on field measurements of long axes of till stones. However, reasons will be advanced for using the modified diagrams in spite of the invalidity of these assumptions.

Reasons for Using the Modified Program

Modified diagrams were prepared using a three per cent area, and were compared with diagrams obtained using the unmodified point density program using a three per cent area. Figure 3 shows the three per cent diagrams produced by both programs for samples 1, 2, 6 and 34-1. For all four samples, the modified program produces smoother, more regular contours. In some cases the unmodified program produces areas containing numberous small highs and lows, whereas the modified program simplifies the contours greatly, eliminating many of the small highs and lows. This can be seen, for example, in sample 34-1 in the southwesterly and southeasterly parts of the diagrams, and in the western part of the diagram for sample 6. If it is assumed that the actual distribution of axes in the population has a simple pattern, then the simplified contours must be regarded as a better approximation to the actual distribution of density in the population.

In certain situations the modified program eliminated maxima produced by the unmodified program that could be shown to be spurious. These may appear when the counting circle is located between two groupings of observations so that it partially overlaps both. One effect of the modified counting procedure is to reduce the effect of points lying near the periphery of the counting circle, hence largely avoiding the above difficulty. This can be seen in sample 1, at the periphery of the projection in the northeastern and southwestern quadrants.

In view of the above-mentioned advantages of the modified program, it was decided to use it to illustrate the fabrics rather than the unmodified program. It must be remembered, however, that in view of the

invalidity of the assumptions concerning error distribution in the measurements, the numbers appearing on the diagrams cannot be taken as accurate estimates of probability. Nevertheless, the numbers probably do represent useful approximations of the probabilities involved. Similarity of the unmodified and the modified diagrams in their major features and relative magnitudes in different areas of the diagram bears this out, and shows that no loss of information is incurred by use of the modified diagrams. Furthermore, comparison of the maximum value appearing on diagrams for different samples using the same size counting circle is believed to give a realistic indication of the relative strength of the modes, that is, of the relative strength of preferred orientation in the data, taking into account a certain amount of expected random variation in the samples. The question of interpretation of the diagrams is discussed more fully in Chapter 4.

Selection of Size of Counting Circle

Following experience gained with the unmodified point density program, modified diagrams were prepared using a three per cent counting circle. Later, modified diagrams were also obtained using a seven per cent circle in order to achieve more of an "averaging" effect. It was felt this would be helpful in the interpretation of the actual direction of preferred orientation represented by each fabric sample.

Figure 4 shows modified diagrams prepared for samples 2, 43, 44 and 45 using counting circles of one, three, five, seven and nine per cent. All diagrams were contoured using a contour interval of e, where e is the "expected" density in the case of a random sample from a

uniform population. For example, for a diagram based on a three per cent counting circle, e is three per cent; when a seven per cent counting circle is used, e is seven per cent. The contours thus provide an indication of the degree of deviation from uniformity in the sample and facilitate comparison of diagrams prepared using counting circles of different size.

It can be seen that the contours become more and more generalized as the counting circle increases in size. The three per cent diagrams give a detailed picture of the sample, indicating unmistakably all major and even minor groupings. The double peak in sample 43 is clearly evident in the three per cent diagram. The seven per cent diagrams do not differ markedly from the three per cent diagrams, but smaller highs are usually assimilated into the larger ones. The two maxima about 60 degrees apart in the northwestern and northeastern quadrants of sample 2 remain separate in the seven per cent diagram. Even the two close maxima in sample 43 remain distinct in the seven per cent plot, but merge into a single elongate high in the nine per cent plot.

The position of a maximum on a point density diagram will be indicated by giving the trend and plunge of the line passing through the density peak. In sample 2, the position of the largest maximum moves from 032 25 to 026 16 as the counting circle increases from three to nine per cent. In sample 43, as counting circle size increases from one to seven per cent, the north-northwesterly-trending maximum moves from 344 32 to 356 29, and the north-northeasterly-trending maximum from 008 12 to 019 07. With a counting circle of nine per cent, a single maximum appears at 005 20. The trend of the principal maximum in sample 44 migrates from 348 in the one per cent plot to 353 in the

five per cent plot, then back to 349 in the nine per cent plot. Plunge of the principal maximum in sample 45 decreases continuously from 15 degrees to six degrees in going from the one per cent to the nine per cent plots, while the trend changes from 303 to 288 and back to 296.

Various rules have been used for determining the optimum size of the counting circle. For instance, Flinn (1958) used a circle of per cent area 100/N where N is the number of observations, so that the expected areas between the contours would be the same regardless of sample size.

Kamb (1959), in constructing ice crystal fabric diagrams of the Blue Glacier, used a different approach. He observed that, in the case of a random sample of N points from a uniform population, the actual number of points, n, falling within the counting circle has a binomial distribution, with mean NA and standard deviation $\sigma = \sqrt{NA(1-A)}$ where A is the area of the counting circle assuming the area of the projection to be unity. In order to avoid wildly fluctuating counts as a result of random sampling, he sets the arbitrary condition that the mean of n be three times its standard deviation. This gives $A = 9/(N + 9)$, or $A = 900/(N + 9)$ per cent. Thus the expected density in a random sample from a uniform population is n or 3σ. Kamb then used contours at 0, 2σ, 4σ, and so on, so that an indication of the degree of deviation from uniformity was obtained. Kamb stated (1959, p.1909): "With the choice of A used here, the difference between the Schmidt and Mellis contouring methods (Flinn, 1958) can lead to no statistically significant differences in the positions of the contours and may therefore be disregarded."

In the case of a sample of size 50, Flinn's rule gives a counting circle size of 100/50 per cent or two per cent. Kamb's rule gives 900/59 per cent or 15.3 per cent. This difference is not surprising. The two workers subjected their diagrams to different analytical procedures, and therefore had different requirements of the diagrams. Clearly, no universal rule can be devised for determining the "best" size of the counting circle; in any given study, the use to which the diagrams are to be put and the information required from them are the ultimate controlling factors in determining the size of counting circle to be used. For example, a small counting circle emphasizes the small-scale variations in the sample. A larger circle will tend to "average out" these local variations while emphasizing the large-scale variations in density within the sample. Thus use of a large counting circle is one way of separating the random and non-random elements in the sample, assuming that the local density variations in the sample are likely to be random while the larger density variations are less likely to be random and more likely to reflect characteristics of the population. This principle of minimizing the effect of the random element is the basis for Kamb's rule for determining counting circle size.

PROCEDURES OF NUMERICAL REPRESENTATION AND ANALYSIS

Review of Methods for Three-Dimensional Orientation Data

Introduction

Flinn (1958, p.527) pointed out the great variety of models that must be considered in analyzing three-dimensional fabric data: "In rock fabrics a single preferred direction is a special case -- more commonly several preferred directions (groups of concentrations) occur; or alternatively a preferred plane or planes (girdles); or concentrations in which the direction is more dispersed in one plane than in that normal to it (partial girdles and elongate concentrations)."

Curray (1956, p.118), in discussing the analysis of two-dimensional orientation data, listed the requirements of a method of analyzing orientation data as: (1) A measure of central tendency or preferred orientation which is independent of origin. (2) A measure of dispersion which is independent of origin. (3) A test of the statistical significance of the results against a model of randomness. (4) A model distribution from which deviations can be tested. Although Curray was concerned only with two-dimensional orientations, the above list is equally applicable to the study of three-dimensional orientation data.

Various statistical procedures have been devised to assist in the performance of one or more of the above steps. Two excellent review articles dealing with many of these procedures are those by Flinn (1958) and Steinmetz (1962). Many of the concepts discussed by Curray (1956)

in relation to two-dimensional orientation data are also relevant.

A brief review of methods that have been used in the treatment of three-dimensional orientation data is given in the following sections. A subdivision of the methods into the following two groups was found convenient. (1) Direct methods: those that treat the data directly in a three-dimensional framework. (2) Indirect methods: those that treat two-dimensional projections of the data. This subdivision is similar to those employed by Flinn (1958) and Steinmetz (1962).

Directed and Non-directed Orientations

This section will define some important concepts and terms to facilitate subsequent discussion of linear quantities. To begin with, any straight line has an <u>orientation</u> in three-dimensional space, usually referred to a conventional system of three mutually perpendicular axes. An orientation may be either directed or non-directed. The orientation of the a-axis of a till pebble is an example of a <u>non-directed orientation</u>. A <u>directed orientation</u> has a <u>sense</u>. Consider, for example, the wind direction indicated by a simple weather vane. This is a directed orientation. In this report the word <u>direction</u> will always denote a directed orientation. In some cases the term "directed orientation" will be used for emphasis instead of "direction". It should be noted that a non-directed orientation may be made into a directed orientation by assigning a sense to it, either arbitrarily or according to some convention.

A <u>vector</u> is a quantity that has an orientation, a sense and a magnitude. If a magnitude is added to the wind direction mentioned above,

the result is the wind velocity, a vector.

Table 1 gives examples of geological quantities having different combinations of the above properties, in both two and three dimensions. Observations of types 2 and 3, that is directed orientations, require methods of treatment that take sense into account, while such methods are not suitable for treating observations of type 1. Similarly, methods suitable for treating non-directed orientations (type 1) are not suitable for the treatment of directed orientations (types 2 and 3). Thus the chief distinction to be made for the purpose of determining suitable methods of treatment is between directed and non-directed quantities.

Indirect Methods

Introduction

Steinmetz (1962, p.805) argued that the tests that analyze three-dimensional data indirectly by means of projections are weaker than tests that analyze the data directly. This is probably true if a suitable direct method is available. However, several years earlier Flinn (1958) had pointed out that all available direct (vectorial) methods were designed to analyze only distributions having a single grouping; as previously noted, it was not until 1964 that Selby dealt analytically with girdle distributions. Accordingly Flinn observed that the vectorial methods were of limited application to rock fabrics, which more commonly showed several modes, or whole or partial girdles, or elongate groupings, to which the vector methods were not applicable. He pointed out that the type of test in which a stereographic diagram is compared by means of the chi-square test with a model random diagram

Table I. Combinations of Properties Observed when Measuring Oriented Quantities in Geology, with Examples.

Type	Number of Dimensions		Properties	
	1	2		
1	pebble axis fold axis intersection of two planes, e.g. bed and joint	trend of horizontal projection of pebble or fold axis trend of cliff, esker, etc. strike of plane	orientation	non-directed
2	direction of remanent magnetization	wind direction direction of dip of cross-bed direction of plunge of axis	orientation + sense	directed
3	direction and strength of magnetic field	horizontal component of wind velocity	orientation + sense + magnitude	

is better able to handle the complexities of a multi-modal, possibly non-normal, distribution (Flinn, 1958, p.527).

Flinn (1958) was concerned only with tests of significance of preferred orientation, and not with the calculation of descriptive statistics. He described several tests of significance, all of which used a stereographic projection, either contoured or uncontoured. Based on these various tests, Flinn derived a procedure for testing a fabric diagram for preferred orientation. Since this procedure incorporates the most useful of the previous tests, only Flinn's method will be described here.

Flinn's Procedure for Detecting Preferred Orientation

The method is based on comparison of the fabric diagram to be tested with random diagrams, that is, diagrams obtained by plotting points drawn at random from a uniform population. The first step is to plot the data on an equal-area projection, and then to contour the point density by the Mellis method by drawing around each point a circle whose area is the area of the projection divided by the number of points plotted. For example, a 100-point plot would be contoured using a one per cent circle. Since a circle is used for contouring, the densities are inaccurate in the outer part of the diagram.

The basis of the tests is the predictability of certain characteristics of random diagrams. Two characteristics were used by Flinn: (1) The total areas contained between the various contours. (2) The frequencies of occurrence of different concentrations, that is, the number of independent and separate areas

of one point per unit area, two points per unit area, and so on. Both of these characteristics of a random diagram may be predicted by means of the Poisson distribution. Flinn also produced 20 random diagrams and verified the predicted values. One advantage of contouring with a circle of area A/N, where N is the number of points plotted and A the area of the diagram, is that the expected density is always the same regardless of the number of points plotted, and therefore the Poisson probabilities of different density values will also be constant, resulting in the same expected areas between given contours. The frequencies of occurrence of different concentrations are proportional to the number of points plotted.

Briefly, application of the test is as follows. The above characteristics are obtained for the fabric diagram under test, and the areas and frequencies compared with those expected in random diagrams by means of chi-square. Any significant deviation is regarded as an indication of a preferred orientation, that is, a departure from complete randomness.

Flinn (1958, p.531) defined three independent ways in which a fabric diagram may show preferred orientation: (1) Grouping: the points may be more concentrated than is to be expected in a diagram drawn at random from a uniform population. (2) Spatial distribution: independently of whether the grouping of the points departs from random expectancy or not, the spatial distribution of points may or may not conform to that expected in diagrams drawn at random from a uniform population. (3) Structural relations: a single concentration or a pattern of concentrations may have a special geometric relation to a structural direction or plane in the rock. This can occur even in a

diagram whose points are both grouped and spatially distributed in a manner to be expected in a diagram drawn at random from a uniform population.

The "area test" is used to test the grouping of the points. The concentration frequencies are used in the "frequency test" to test the spatial distribution of the points against randomness. One final technique that Flinn calls the "two-cell zonal test" completes the procedure. This consists of drawing a circle concentric with the projection that divides the projection into two equal areas or cells. In a random fabric, the same number of points would be expected in each cell, and the observed numbers are tested against this expectancy using chi-square. Thus if the fabric is first rotated to an appropriate position, the test may be used to detect either a girdle or a point concentration. In Flinn's procedure, this test is used as one possible test for particular patterns after a non-random grouping or spatial distribution has been detected. It is also used to dectect a point concentration or girdle related to a particular structural direction in the rock, by first rotating the fabric until the structural direction to be tested lies at the center of the projection.

The chief purpose of Flinn's procedure is the detection of a significant deviation from randomness in a fabric diagram. The two-cell zonal test can be used to test for certain patterns. The procedure is not concerned with the calculation of descriptive statistics, and accordingly provides no powerful means of comparing fabric patterns.

The method has the advantage that a deviation from randomness can be detected before the kind of preferred orientation (the fabric

pattern) is known. The method is well suited to the detection of very weak preferred orientations (Flinn, 1958, p.531). According to Flinn (1958, p.532, 533) it takes about 12 minutes to contour a 100-point diagram using his method, and about 100 minutes per 100 points plotted to obtain both the areas and the frequencies of the concentrations. Before the two-cell zonal test can be used, the fabric must be rotated, but does not need to be contoured.

Reproducibility as a Test of Significance

Flinn (1958, p.526) listed three basic kinds of test of significance of preferred orientation: (1) Similarity of successive samples. (2) Comparison with preferred orientation. (3) Comparison with randomness. Method 3 is that employed by Flinn, and also by Watson (1956b) in his test of the vector resultant, R. A few words will be said here about method 1.

Flinn (1958, p.526) argued that comparison of successive samples as a test of significance of a fabric pattern requires the knowledge or assumption that the fabric in question is homogeneous. If the homogeneity of the fabric were not first established, differences between successive fabric samples could be interpreted as indicating a lack of significance of the fabric characteristics involved, or as indicating spacial variation of the fabric.

However, Kauranne (1960), in till fabric studies in Finland, tested the reproducibility of fabric samples by the following procedure. In a set of 200 observations of pebble axes, all the odd-numbered observations were assigned to sample A, while all the even-

numbered observations were assigned to sample B, thus producing two samples of 100 measurements each. The fabric diagrams of samples A and B were then compared to test the reproducibility of the fabric (Kauranne, 1960, p.91 and Table 1, p.90). As a result of this procedure, any spatial variation in the fabric will appear in both diagrams A and B. The two samples A and B are independent duplicate samples of the same fabric. If similarities are found, they can be interpreted as indicating non-randomness. Any differences must be the result of random sampling, since they are not due to spatial variation of the fabric. This technique could be made yet more diagnostic by taking larger samples and assigning the measurements to three or four groups instead of two.

The above method could be used to establish the minimum sample size required to define the fabric with any desired degree of reliability. The method would be particularly valuable in the case of, for example, a complex fabric not amenable to direct mathematical treatment. Kamb (1959, p.1900), in discussing ice-crystal fabrics, emphasized the importance of reproducibility as a test of significance of fabric properties.

Reproducibility in duplicate samples taken during the present study to establish the reliability of samples of 50 long axes is discussed in Chapter 4. In evaluating the reproducibility of sample characteristics in order to determine whether or not the sampled population is non-uniformly distributed, it is important that examples of random samples from uniform populations also be studied. These illustrate the degree of reproducibility, or non-reproducibility, to be expected if the population is uniform, and thus provide a basis for

recognizing a significant degree of reproducibility.

Direct Methods

Vector Summation Method

Fisher (1953) suggested a probability distribution as a basis for the statistical treatment of <u>vectors</u> scattered about a mean direction. Watson and Irving (1957) employed Fisher's distribution in a detailed development of a suitable statistical procedure for the treatment of observations of the direction of magnetization of rocks. Fisher (1953) suggested that those directions will, when regarded as points on a unit sphere, be distributed over the sphere in accordance with the function

$$C.\exp(K\cos\phi)$$

where K is a precision parameter, ϕ is the angle between the polar vector (the centre of the distribution) and the observation vector, and C is a constant. The density thus has axial symmetry about the polar axis, and attains a maximum at the pole and a minimum at the anti-pole. When K=0 the density is uniform over the sphere. When K is large the density is confined to the region about the pole and has approximately a spherical normal destribution in this region (Watson, 1956a, p.153).

If (l_i, m_i, n_i) are the direction cosines of the i-th observation of N observations, and (l,m,n) are the direction cosines of the maximum likelihood estimate of the polar vector, then Fisher (1953) has shown that

$$l = \frac{\Sigma l_i}{R}, \qquad m = \frac{\Sigma m_i}{R}, \qquad n = \frac{\Sigma n_i}{R}$$

where $R^2 = (\Sigma l_i)^2 + (\Sigma m_i)^2 + (\Sigma n_i)^2$. Thus R is the length of the vector resultant of all the vector observations, and (l,m,n) are the direction cosines of this vector resultant (Watson, 1956a, p.154). Fisher also showed that the best estimate k of K, the precision parameter of the population, is given by, for K greater than 3,

$$k = \frac{N - 1}{N - R},$$

where N is the size of the sample (Watson and Irving, 1957, p.292).

Watson (1956a) gave approximate tests for the equality of the K's and polar directions for any number of populations. These tests of significance were elaborated by Watson and Williams (1956). Watson (1956b) gave a table of significance points for a test of K=0 using R, the length of the vector resultant of the observation vectors.

Watson and Irving (1957) described the application of the above methods to paleomagnetic data. Since the method they described is the chief method used in the present study, it will be delineated in some detail below. After calculating the best estimates of the polar direction and precision parameter, they applied Watson's (1956b) test of randomness. This test is based on the argument that, "Given a sample of size N, the length R of the vector resultant will be large if the sample shows a preferred direction and small if it does not. Assuming that there is truly no preferred direction (that is, K=0), a value R_o, say, may be calculated which will be exceeded by R with any stated probability." (Watson and Irving, 1957, p.292). Watson (1956b) has tabulated R_o for N=5(1)20 and probabilities 0.05 and 0.01. He has

also shown that for $N > 20$, R_o is successfully approximated by the expression

$$\sqrt{\frac{N\chi_3^2}{3}}$$

where χ_3^2 stands for a chi-square variate with three degrees of freedom.

It should be carefully noted that the above test of randomness was designed to test a set of unit vectors whose sample space corresponds to a sphere. The importance of this fact will be discussed in subsequent sections.

The next step in the analysis of Watson and Irving (1957) is the comparison of the precision parameters of different populations. For two samples consisting of N_1 and N_2 observations that give precision parameters k_1 and k_2, the ratio k_1/k_2 has the F-distribution with $2(N_2 - 1)/2(N_1 - 1)$ degrees of freedom, and may be used to test the hypothesis that $K_1=K_2$. Values of k_1/k_2 far from unity suggest that $K_1 \neq K_2$. For several populations, the ratio of the largest to the smallest k may be referred to tables of the maximum F-ratio (Hartley, 1950; David, 1952), in order to test the hypothesis that K is constant over the populations. Next, the accuracy of the calculated mean direction is estimated (Watson and Irving, 1957, p.292) as follows. If c is the cosine of the angle between the calculated mean direction and the true mean, Fisher has shown that this cosine will be less than c with probability P where

$$c = 1 - \frac{N-R}{R}\left\{\left(\frac{1}{P}\right)^{\frac{1}{N-1}} - 1\right\}$$

"To test whether the true mean directions of p populations are identical, the statistic

$$\frac{2(\Sigma N_i - P)}{2(p-1)} \cdot \frac{\Sigma R_i - R}{\Sigma N_i - \Sigma R_i}$$

may be referred to the F-ratio tables with 2(p - 1) and 2(N_i - p) degrees of freedom. It is supposed that the sample from the i-th population contains N_i specimens and has a resultant of length R_i and that R is the length of the vector sum of resultants of the separate samples. Again, large values of the statistic suggest that the assumption of identical true mean directions is false because the algebraic sum of the sample resultants R_i will then be much greater than the length of their vector sum, R." (Watson and Irving, 1957, p.293).

Watson and Irving then described a method of testing the fit of the observations to Fisher's distribution, using chi-square. Finally they presented a technique that may be used when observations are made at several sites, and that permits calculation of within-site and between-site precision parameters. This technique can also be used to determine, on the basis of initial test samples, the optimum sampling pattern to define an overall mean direction with a given accuracy (Watson and Irving, 1957, p.298).

The above method of treating sets of three-dimensional vectors is compared with the required operations previously listed. The R test for randomness serves the purpose of establishing the presence or absence of a preferred orientation. The method is designed to be applied when the basic form of the distribution is known to be unimodal and approximately spherical normal. Fit of the data to a spherical normal distribution can be tested. If the fit is very poor, or if the

distribution is bi- or multi-modal, the method cannot be applied as described. The method provides statistics describing the location of the distribution and its shape; it also allows comparisons to be made of different distributions. Hence the method takes care of all required operations except determination of the kind of preferred orientation, that is, the basic form of the distribution. It is applicable only if the distribution is known to be unimodal and approximately spherical normal.

Watson (1960) extended the above techniques to construct a test for coplanarity of population means, a property sometimes of importance in geophysical studies. He further adapted the method for the study of folded strata (Watson, 1965). In the latter application, unit vectors normal to various portions of a surface having nearly cylindrical folds are distributed on the sphere in the form of a partial girdle. This differs from a girdle in lacking rotational symmetry (Watson, 1965, p.193). The problem of girdles has been dealt with by Selby (1964).

In all the situations just mentioned, the form of the distribution must be known or determined before the appropriate method can be used, and if the pattern is complex (for example, a point concentration and a girdle may both be present) the methods are not directly applicable. Furthermore, the methods were designed to deal with observations of directed quantities, and great caution must be used in applying them to a set of non-directed axes.

Eigenvector Method

A technique designed specifically to treat a set of non-directed axes was presented by Scheidegger (1965). As in the previous method, the procedure is valid only for unimodal distributions. Scheidegger postulates a distribution of the form

$$P = \text{const.}\exp(k.\cos^2\phi) \text{ ,}$$

where P is the probability at an angular distance ϕ from the central axis of the distribution, and k is a precision parameter. For k=0 the distribution is uniform. The distribution differs from that of Fisher (1953) only in having $\cos^2\phi$ in place of $\cos\phi$. This makes Scheidegger's distribution a slightly different shape from Fisher's. More important, it means that the former is periodic in ϕ with a period of 180 degrees (Scheidegger, 1965, p.C165), whereas the latter has a period of 360 degrees. Stated differently, this means that whereas Fisher's distribution has as its sample space the whole sphere, Scheidegger's distribution is restricted to a hemisphere. The reason for this difference is as follows. Fisher sought to describe the distribution of the directions of directed lines, whose sample space is the whole sphere. Scheidegger's purpose, on the other hand, was to describe the distribution of the orientations of non-directed lines, whose sample space is a hemisphere since any orientation may be represented in a given hemisphere.

The problem of finding the mean and variance of a given sample may be regarded as the problem of fitting an appropriate hypothetical distribution to the sample. The distribution is located in such a way as to maximize the probability of the sample. Scheidegger's method

operates as follows. Let P_i be the probability of the i-th axis in a sample of N axes, and let the smaller angle (that is, less than 90 degrees) between the i-th axis and the central direction of the distribution be ϕ_i. The probability of the sample is given by the product of the probabilities of the individual axes that make up the sample. That is

$$P_s = \mathrm{Prod}(P_i)$$

where P_s is the probability of the sample and $\mathrm{Prod}(P_i)$ means the product of all the P_i. Writing this in terms of Scheidegger's density function,

$$\begin{aligned} P_s &= \mathrm{Prod}(\mathrm{const.exp}(k.\cos^2\phi_i)) \\ &= \mathrm{const.exp}(\Sigma\, k.\cos^2\phi_i) \\ &= \mathrm{const.exp}(k.\Sigma \cos^2\phi_i) \quad . \end{aligned}$$

Clearly, P_s will be a maximum when $\Sigma \cos^2\phi_i$ is a maximum. If (l,m,n) are the direction cosines of the central direction of the distribution, and (l_i, m_i, n_i) the direction cosines of the i-th axis in the sample, then

$$\cos \phi_i = l.l_i + m.m_i + n.n_i$$

and

$$\Sigma \cos^2\phi_i = a_{11}l^2 + a_{22}m^2 + a_{33}n^2 + 2a_{12}lm + 2a_{13}ln + 2a_{23}mn$$

where

$$\begin{aligned} a_{11} &= \Sigma\, l_i^2 \\ a_{22} &= \Sigma\, m_i^2 \\ a_{33} &= \Sigma\, n_i^2 \\ a_{12} &= a_{21} = \Sigma\, l_i m_i \\ a_{13} &= a_{31} = \Sigma\, l_i n_i \\ a_{23} &= a_{32} = \Sigma\, m_i n_i \end{aligned}$$

Thus the problem is to maximize the above quadratic in l, m and n under the condition that $l^2 + m^2 + n^2 = 1$ (since l, m and n are direction

cosines). The solution to this classical problem is that (l,m,n) are the direction cosines of the eigenvector of the matrix a_{ik} corresponding to the largest eigenvalue v_1. This eigenvalue is the corresponding value of $\sum\cos^2\phi_i$. Hence

$$\frac{v_1}{N} = \frac{\sum\cos^2\phi_i}{N}$$

which is the mean square of the cosines of the deviation angles. This mean square cosine may be converted to an angle, which may be regarded as a standard scattering angle. The direction of the eigenvector corresponding to the smallest eigenvalue is the direction in which $\sum\cos^2\phi_i$ is a minimum. A corresponding scattering angle may be calculated.

The method provides an estimate of the mean of a sample, and an estimate of scattering. The distribution must be known to be unimodal and to conform approximately to the distribution const.exp(k.$\cos^2\phi$). It would probably be possible to construct a test for randomness using the maximum value of $\sum\cos^2\phi_i$, and tests for comparing distributions using the statistics given by this method, but it appears that such tests have not yet been devised.

Further Discussion of Fisher's and Scheidegger's Distributions

It is interesting to compare Scheidegger's (1965) method of locating the mean of a set of directions to that of Fisher (1953). If the same approach of maximizing the sample probability is taken using Fisher's distribution, the problem of locating the mean of a sample reduces to that of finding the direction that maximizes $\sum\cos\phi_i$. But

$\cos \phi_i$ is simply the component in the reference direction of a unit vector in the direction of the i-th observation, and the sum of the components in a given direction of a set of vectors is a maximum when that direction coincides with the direction of the resultant of the vectors. Hence Fisher's solution is obtained: that the best estimate of the mean of a set of directions is the direction of the vector sum of a set of unit vectors in the given directions.

Mathematically the above can be stated as follows. If (l,m,n) are the direction cosines of the required mean direction, and (l_i,m_i,n_i) the direction cosines of the i-th observed direction, then

$$\cos \phi_i = l.l_i + m.m_i + n.n_i$$

and

$$\Sigma \cos \phi_i = l.\Sigma l_i + m.\Sigma m_i + n.\Sigma n_i = F$$

and the problem is to maximize F under the condition $l^2 + m^2 + n^2 = 1$. This condition may be used to eliminate one variable from F, making it a quadratic in two variables. The solution for the maximum is then obtained by equating each of the two partial first derivatives to zero. This gives

$$l = \frac{\Sigma l_i}{R}, \quad m = \frac{\Sigma m_i}{R}, \quad n = \frac{\Sigma n_i}{R},$$

where $R^2 = (\Sigma l_i)^2 + (\Sigma m_i)^2 + (\Sigma n_i)^2$. Thus (l,m,n) are the direction cosines of the resultant of unit vectors in the given directions, which is the result obtained by Fisher (1953).

If one attempts to follow a similar procedure, again using Fisher's distribution, for a set of non-directed axes, it is necessary to use $|\cos \phi_i|$ (the absolute value of cosine ϕ_i), since a non-directed axis always makes an angle of less than 90 degrees with any given direction. Now the problem is to maximize $|\cos \phi_i| = |l.l_i + m.m_i + n.n_i|$.

This, however, cannot be expanded as was $\Sigma \cos \phi_i$, and therein lies the real advantage of Scheidegger's distribution. Since the latter distribution uses $\cos^2\phi_i$ instead of $\cos \phi_i$, it makes no distinction between $\cos \phi_i$ and $-\cos \phi_i$, and hence is suitable for non-directed axes. Furthermore, $\Sigma \cos^2\phi_i$ may be expanded as previously indicated, allowing the straightforward eigenvector solution. In many computer systems subroutines are available for calculation of eigenvalues and eigenvectors of a real symmetric matrix, such as the one employed in this procedure.

One of the computer programs written during the course of this study would, with a simple modification, make possible a graphical solution to the problem of maximizing $\Sigma \left| \cos \phi_i \right|$, thereby permitting the treatment of non-directed orientations using a model distribution having the same shape as Fisher's distribution. Such a procedure would still, however, be limited to unimodal distributions.

The Treatment of Non-Directed Orientations

Parts of this report are concerned with evaluating a published procedure for applying the vector summation method described in a foregoing section to the orientations of the a-axes of till pebbles, and with devising an improved procedure. It seems logical, therefore, to consider at this stage the more general problem of applying the vector summation method to any set of non-directed orientations, as a basis for later discussions.

Fisher's vector summation method was designed for estimating the mean of a set of <u>directed</u> orientations <u>grouped about a single</u>

direction on the sphere. Any orientations to which the method is to be applied must be in the form of vectors or directed orientations. To make a non-directed orientation into a directed orientation it is necessary to assign a sense, that is, to choose one end of the axis or the other. This fact was pointed out by Steinmetz with reference to c-axes of quartz grains in metamorphic rocks: "The c-axes could become vectors only if a sense were assigned by some external criterion." (Steinmetz, 1962, p.801). It was also recognized by Scheidegger (1965, p.C164): "A vector is a directed quantity, an axis is not. Hence, in order to assign a vector to each axis, it is necessary to make a convention with regard to the side of the axes in which the corresponding vector is supposed to point."

The only method of assigning sense to a number of axes while preserving the characteristics of the original distribution of the axes is to cut the distribution in half using a plane passing through the origin of the reference axes. In other words, instead of considering the distribution as occupying a sphere, one considers only that part of the distribution that lies in a given hemisphere. It should be observed that any plane will cut a set of non-directed axes into two identical hemispheres. It is important to remember that, while such a hemispherical distribution completely defines the original spherical distribution and therefore adequately represents it, it is not identical to it, since the original distribution consisted of two such hemispheres.

Thus if a set of non-directed orientations is unimodally distributed, that is, the orientations are grouped about a single orientation, it is usually possible to obtain satisfactory results by consider-

ing only that part of the distribution that lies in a selected hemisphere, assigning unit vectors in the observed directions in the hemisphere, and then utilizing Fisher's vector summation technique to locate the best estimate of the mean. It is only necessary that sufficient care be taken in selecting the hemisphere to be used.

Distributions of non-directed orientations that are not unimodal, that is, in which the orientations are grouped about two or more centres, cannot be handled in this way. The vector summation method can be applied only if the observed orientations are divided into a number of groups each of which has a unimodal distribution.

The Question of Reference Planes

Much of the confusion that has characterised the literature concerning geological applications of the various methods of treating orientation data has two chief causes: (1) The failure of workers applying the methods to recognize the differences between different kinds of orientation data, particularly the distinction between directed and non-directed orientations and the limitations imposed on the methods by this fundamental distinction. (2) The failure of workers applying the methods to recognize the distinction between the several different functions of reference planes used in the treatment of orientation data.

The different kinds of orientation data commonly used in geology were described previously; this section will describe three separate functions performed by reference planes.

1 When orientations are measured in the field they are referred to the horizontal plane; this is the first reference plane. Directions

in this plane are usually referred to true or magnetic north. These are conventions, adopted for convenience and to ensure reproducibility and compatibility of observations.

2 Measurements of orientations in two dimensions can be plotted directly on a two-dimensional diagram. However, before orientations measured in three dimensions can be represented on a two-dimensional diagram they must be projected in some way onto a selected plane. This projection plane is the second reference plane. It need not be the same plane as that used as a reference for the field notes; the data may be projected onto any plane. However, use of the horizontal plane as a projection plane offers certain advantages: (a) the data may be related directly to compass directions, (b) directions of dip are readily apparent, and (c) orientations measured using a horizontal reference plane are most easily plotted when the projection plane is also horizontal.

3 As described previously, it is possible to use certain treatment methods designed for directed quantities in the treatment of non-directed quantities provided appropriate care is taken to select a suitable origin. In the case of two-dimensional data, this origin is a line in the data plane; when the observed quantities are three-dimensional, this origin takes the form of a plane. This is the third reference plane. It has nothing whatever to do with either the measurement reference plane or the projection plane used for display purposes. All three planes are distinct and completely independent. There is no reason, for example, why three-dimensional data cannot be projected onto

a horizontal plane for display purposes, while being mathematically treated using a vertical plane or any other plane as a reference for assigning sense to the orientations.

Before the advent of electronic computers, a worker wishing to adjust orientation measurements on a set of axes so that the specified ends of the axes would fall in some hemisphere other than the conventional one below the horizontal would have used a graphical procedure. The data would have been plotted on a horizontal projection, and, with the use of a stereonet, adjusted graphically so that the horizontal projection would, in effect, be transformed into a projection of the same data on the desired plane. The data would then be "projected back" into three dimensions by reading off the new trends and plunges. This is probably the reason why the analysis of data using other than the conventional lower hemisphere is inseparably bound, in the minds of some workers, to the procedure of "rotating" the data to form a new projection. However, the new projection is not required for the mathematical treatment: it is only necessary that the three-dimensional data be adjusted to the desired hemisphere. This can be accomplished directly without the use of a new projection and without rotating the data.

The orientation of a pebble a-axis is normally measured as trend and plunge with the plunge measured downward from the horizontal. Thus the resulting values of trend and plunge actually indicate the direction of the <u>downward-pointing</u> end of the axis. It should be remembered, however, that the use of the horizontal plane as a reference, and the

measurement of plunge downward rather than upward from the reference plane are merely conventions, established to permit unambiguous reporting of three-dimensional orientations. There is no implication that the axis is a downward-pointing directed quantity. Treatment of the measurements directly as downward-pointing directed quantities constitutes use of the horizontal plane as a reference plane for assigning sense to the axes, and this may or may not be a suitable way of treating the particular set of data.

Summary

Flinn's (1958) procedure is designed to detect preferred orientation and test the relationship of preferred orientations to particular directions. It is specifically intended for use in marginal cases in which the preferred orientation is weak. The method was not designed to determine the preferred orientation or any accurate estimate of the strength of preferred orientation.

Scheidegger's (1965) procedure was designed to determine the preferred orientation and amount of scattering in a set of non-directed axes. Its disadvantages are: (1) It does not provide precise tests for comparing samples based on the statistics produced. (2) It treats only unimodal distributions having an axially symmetrical grouping.

Watson and Irving's (1957) procedure was designed to treat vectors, and does provide both descriptive statistics and precise tests using these statistics for comparing samples. It also requires unimodal distributions with axial symmetry.

Hence the most complete procedure is provided by Watson and

Irving's method. However, just as Scheidegger's method was not designed to treat vectors, that of Watson and Irving was not designed to treat non-directed orientations, and the application of it to, for example, till fabrics must not be done without careful consideration of the applicability of each step of the procedure.

It is important to recognize that each of the above methods was designed specifically for either directed or non-directed orientations, and that a given method may not be directly applicable to different kinds of orientation data. Important too is the recognition of the three functionally distinct reference planes used in the treatment of orientation data: (1) The horizontal reference for measurements. (2) The projection plane used for graphical display of the data. (3) The plane used for assigning sense to non-directed axes if they are to be treated by a method requiring directed lines.

Methods Examined for Applicability to Till Fabric Data

Previous Application to Till Fabrics

Only one of the above-described methods has previously been used in the treatment of till fabrics: the method described by Watson and Irving (1957) for the analysis of paleomagnetic vectors was experimentally adopted by Andrews and Shimizu (1966) for the analysis of till fabrics. They used a FORTRAN IV program which assigned unit vectors to the downward-pointing ends of the till stone long axes as specified by their azimuth and plunge, and then computed the direction and

magnitude of the vector resultant. In addition, it calculated an estimate, k, of Fisher's precision parameter K using the formula given by Watson (1956a):

$$k = \frac{N - 1}{N - R}$$

where N is the number of observations and R the length of the resultant vector. The program also provided the 95 per cent confidence radius for the calculated mean direction, and the magnitude, R, of the resultant vector. The analytical prodedure was described by Andrews and Shimizu (1966, p.156) as follows:

"The approach to the analysis of the computer results is best seen through the series of progressive steps listed below:

"(1) Obtain till fabric data and feed it into the computer program.

"(2) Test the hypothesis that the three-dimensional distribution is random by evaluating R against an acceptable confidence level, e.g., 90, 95 or 99 per cent.

"(3) Accept or reject the null hypothesis; if "accept" the analysis must stop, if "reject" the analysis proceeds to (4).

"(4) Plot the values for orientation and dip as well as the limits of the confidence area which has been determined prior to the analysis. This involves rerotation if rotation has already been applied to the observations.

"(5) Ascertain whether the sample approximates a spherical-normal distribution, the <u>lower limit of acceptance</u> of this hypothesis being $\underline{k} \geq 3$. If it does not approximate a spherical-normal distribution, then the analysis must cease or an attempt be made to improve the value of <u>k</u> by rotation of the data. Implicit in this <u>lower limit of accep-</u>

tance is the fact that, with k equal to 3 for 95 per cent of the time, observations will lie within a radius of $\pm$ 90 degrees about the mean vector. As k increases this limit becomes smaller.

"(6) Examine other samples in the same manner and compare the results by use of the maximum F test (see above). If variance is homogeneous proceed to (7).

"(7) Use the F test to ascertain if sample orientations belong to the same parent population.

"(8) Use the results in an analysis of variance design to determine precision parameters ω and β (Watson and Irving, 1957)."

The three till fabrics analyzed by Andrews and Shimizu (1966) were processed using the data as measured in the field, processed again after the data had been rotated through 30 degrees about a horizontal axis and again after rotation through 90 degrees. It is not clear what axis of rotation was used, but it seems to have been a horizontal east-west line. Andrews and Shimizu observed that an increase in the calculated R and k values resulted from both of these rotations, the maximum values being obtained with the 90 degree rotation. They concluded that such a 90 degree rotation should be used to analyze till fabrics.

In order to take care of pebbles lying in the B-lineation (that is, transverse to the ice-movement direction), Andrews and Shimizu (1966, p.160) suggested that, in order to keep sample numbers constant, "10 per cent of the observations lying $\pm$ 90 degrees from the preferred orientation after a 90-degree rotation has been completed, be omitted from the calculations and the results rerun."

This method was again used by Andrews and King (1968), with the

addition of some calculations based on Watson and Irving's (1957, p.297) analysis of dispersion table for the calculation of within- and between-site precision parameters and the determination of optimum sampling patterns. Andrews and King (1968) regarded the scatter of pebble axes in a fabric sample and the variation in the mean directions of different fabric samples as equivalent to Watson and Irving's within-site and between-site dispersion respectively.

Andrews and Shimizu's (1966) procedure was used by Cowan (1968) and Caine (1968). These works added nothing to the procedure itself and will be discussed in a subsequent section.

First Method Examined

Since the method employed by Andrews and Shimizu (1966) appeared to offer certain advantages over conventional, strictly visual methods of evaluating and comparing till fabric diagrams, it was decided to investigate its applicability to the data used in the present study. The descriptive statistics obtained were examined in relation to graphical displays of the same fabric data (the point density diagrams obtained previously) in order to determine how meaningful and how useful the calculated statistics were. This was regarded as a necessary precaution before using the statistics in various tests concerning the similarity of population means and variances.

An attempt was made to calculate meaningful statistics to represent the fabric data using the approach described by Andrews and Shimizu (1966) with two modifications. Firstly, an improved computer program was written that avoided the necessity of rotating the fabric

data and, secondly, another computer program was written in an attempt to eliminate the arbitrary selection of a reference plane for use in the calculation of the statistics. These two modifications are described in detail below.

As discussed in the section on directed and non-directed orientations, before any set of non-directed axes can be analyzed by a method designed to treat vectors, the axes must be assigned a sense by choosing a reference hemisphere. Frequently, a set of non-directed axes grouped about a single direction is transformed into a set of vectors in order that the resulting set of vectors may be treated as having a Fisher distribution. The closest approximation to a Fisher distribution is obtained when sense is assigned to the axes using a dividing plane normal to the central tendency of the grouping.

Andrews and Shimizu used a computer program that assigned vectors to the axis orientations as specified. Since the axis orientations were specified by their trend and plunge (measured downward from the horizontal), the resulting set of vectors necessarily occupied the lower hemisphere. Hence the resulting set of vectors had approximately a Fisher distribution only if the axis distribution had its central tendency vertical. For this reason, it was necessary for Andrews and Shimizu to rotate each set of axes until its central tendency was nearly vertical.

For the purpose of the present study, a computer program was written that calculated the same statistics as those calculated by the program used by Andrews and Shimizu, with the addition of the vector magnitude, L, equal to R/N. However, instead of assigning vectors to the axes directly as specified by their trend and plunge so that the

resulting set of vectors occupies the lower hemisphere, the program is capable of assigning vectors so that they lie in any desired hemisphere. The desired hemisphere is specified by the trend and plunge of its axis, that is, the normal to its equatorial plane. Thus, the axis of the hemisphere may be made to coincide with the central tendency of the distribution. This achieves the desired relationship between the data and the reference hemisphere without the tedious and time-consuming process of rotating the fabric data. This program is referred to as the "mean vector program". Details of it may be found in Appendix D.

The second modification to the method described by Andrews and Shimizu (1966) concerns the selection of the axis about which the data was rotated, and the angle of rotation. In terms of the new program described above, this means the selection of the position of the reference hemisphere. Andrews and Shimizu recommended a rotation of 90 degrees, and in their examples used an east-west axis of rotation. No reason was advanced for choosing this axis of rotation, other than that it increased the tightness of grouping when the data were projected onto a horizontal plane. Only two rotations were tried: a rotation of 30 degrees about an east-west axis, and a rotation of 90 degrees about and east-west axis. The 90 degree rotation was considered to be the better, since it produced a horizontal projection with a greater degree of cluster. No other rotations were considered.

In the present study, selection of the position of the reference hemisphere corresponds to the selection of a suitable rotation in the procedure of Andrews and Shimizu. One approach to the problem of

selecting a good position for the reference hemisphere would have been to try two or three possible positions and then choose the one that produced a set of vectors with the highest degree of cluster. However, this seemed like something of a hit-and-miss approach, and it was decided to attempt to devise a method of locating the optimum position for the reference hemisphere. The basic aim was to produce a set of vectors having a distribution most closely approximating a Fisher distribution; in practice this was done by choosing the set of vectors that had the highest degree of cluster. Vector magnitude (Curray, 1956, p.119-120) was chosen as a numerical indication of the degree of cluster of a given set of vectors. A computer program was then written that would, in effect, place the reference hemisphere successively in 333 different positions, and for each position would calculate the vector magnitude of the resulting set of unit vectors. The 333 values of vector magnitude were then printed on a stereographic projection, each value being located at the point corresponding to the pole of the reference hemisphere used in its calculation. The resulting plot thus provided a graphical display of the variation in the tightness of grouping of the vectors as the reference hemisphere was placed in different positions. This plot was roughly contoured and the peak value located. The position of this peak value indicated the position of the reference hemisphere that produced a set of vectors having the highest vector magnitude, that is, the highest degree of cluster. This position was then specified when calculating the statistics using the program described above.

This approach of finding the optimum position of the reference plane based on the highest degree of cluster (or minimum variance) was

also suggested by Cowan (1968, p.1149) and Andrews and King (1968, p.443) but was not employed by them.

Chayes (1954) put forward the principle of using the "minimum variance origin" in the treatment of non-directed orientations in two dimensions. The method described above is a direct extension of that principle to three dimensions.

This program for producing a plot of vector magnitude variation is referred to as the "vector magnitude program". Details of it may be found in Appendix D.

Thus the procedure initially used to obtain descriptive statistics may be summarized as follows. The data for each sample were processed using the vector magnitude program. The diagram produced was roughly contoured and the trend and plunge of the peak position determined using a Schmidt net. This trend and plunge were then punched onto a card and subjected, along with the fabric data, to treatment by the mean vector program. The statistics obtained using this method are here called "whole-sample" statistics and are listed in Table 2. The mean direction calculated by this method will be referred to as the "whole-sample mean direction", or simply, the "whole-sample mean". The significance of these results will be discussed in the next section.

TABLE 2. Results of first method: characteristics of vector magnitude diagrams and whole-sample statistics calculated using reference plane giving maximum vector magnitude. "Plane" = plane giving minimum vector magnitude, specified by dip direction and dip; N = sample size; k = estimate of K, precision parameter of population; R = length of resultant vector; L = vector magnitude, R/N.

		VECTOR MAGNITUDES DIAGRAM							STATISTICS						
STA		MAX		MIN			PLANE		MEAN		CONF.RAD.				
NO.	N	TR	PL	L%	TR	PL	DIR	DP	TR	PL	95%	99%	K	R	L(%)
1	50	026	15	28	250	74	070	06	017	09	12.6	15.7	3.59	36.4	72.7
2	50	030	17	31	168	66	348	24	025	14	12.2	15.2	3.76	37.0	73.9
3	62	050	10	31	256	68	076	12	049	15	14.1	17.6	2.65	39.0	62.8
4	50	304	11	28	209	62	029	28	304	05	14.9	18.6	2.84	32.8	65.5
5	52	182	03	32	288	78	108	12	184	03	14.3	17.9	2.91	34.5	66.3
6	50	078	13	30	305	85	125	05	074	07	12.5	15.6	3.63	36.5	73.0
7	51	045	00	18	205	83	025	07	046	06	11.1	13.9	4.24	39.2	76.9
8	50	345	35	24	201	30	021	60	332	47	12.2	15.2	3.76	37.0	74.0
9	50	031	00	17	153	72	333	18	037	10	10.8	13.5	4.48	39.1	78.2
10	51	042	09	21	190	79	010	11	041	07	13.8	17.2	3.10	34.9	68.4
11	50	249	23	27	104	80	284	10	249	13	13.3	16.7	3.30	35.2	70.3
12	49	029	13	20	260	82	080	08	028	01	11.4	14.3	4.20	37.6	76.7
13	58	156	06	23	268	74	088	16	152	05	11.6	14.5	3.61	42.2	72.8
17	50	282	26	30	025	60	205	30	277	18	14.6	18.3	2.90	33.2	66.3
18	50	330	18	20	176	59	356	31	327	27	10.2	12.7	4.96	40.2	80.3
20	50	328	02	28	164	80	344	10	323	05	14.4	18.0	2.97	33.5	67.0
21	50	029	08	32	269	70	089	20	024	07	14.0	17.5	3.08	34.1	68.2
22	50	256	09	26	179	40	359	50	260	01	12.2	15.3	3.74	36.9	73.8
23	50	207	02	39	119	53	299	37	027	00	16.0	20.1	2.58	31.1	62.1
24	50	022	13	22	170	63	350	27	021	21	10.1	12.6	5.04	40.3	80.6
25	50	000	10	36	108	38	288	52	000	14	15.9	19.8	2.62	31.3	62.6
26	50	358	08	27	227	62	047	28	346	12	10.1	12.6	5.03	40.3	80.5
27	50	025	03	32	111	65	291	25	025	02	13.8	17.2	3.14	34.4	68.8
28	50	357	07	32	252	76	072	14	357	09	14.1	17.7	3.04	33.9	67.8
29	50	044	06	28	235	81	055	09	046	05	15.5	19.3	2.70	31.9	63.7
30	50	125	31	36	024	28	204	62	133	32	14.1	17.7	3.04	33.9	67.8
31	50	294	17	33	137	68	317	22	294	14	16.8	21.0	2.45	30.0	59.9
32	50	138	00	35	248	59	068	31	141	01	15.2	19.1	2.75	32.2	64.4
33	50	332	27	31	152	63	332	27	332	27	14.7	18.4	2.87	33.0	65.9
34	100	053	07	32	235	83	055	07	052	08	9.6	11.9	3.21	69.1	69.1
34-1	50	068	10	33	314	73	134	17	061	07	14.0	17.6	3.07	34.1	68.1
34-2	50	031	05	29	174	76	354	14	034	06	12.4	15.5	3.66	36.6	73.2
35	100	031	03	31	115	37	295	53	026	04	9.4	11.7	3.30	70.0	70.0
35-1	50	031	05	34	112	32	292	58	028	04	13.7	17.1	3.19	34.7	69.3
35-2	50	202	10	28	119	42	299	38	024	03	13.1	16.4	3.36	35.4	70.8

TABLE 2 (continued)

		VECTOR MAGNITUDES DIAGRAM							STATISTICS						
STA		MAX		MIN			PLANE		MEAN		CONF.RAD.				
NO.	N	TR	PL	L%	TR	PL	DIR	DP	TR	PL	95%	99%	K	R	L(%)
36	50	038	39	24	218	61	038	29	028	27	11.4	14.3	4.14	38.2	76.3
37	100	270	03	32	173	71	353	19	268	03	11.5	14.4	2.51	60.6	60.6
37-1	50	045	00	32	173	70	353	20	043	03	14.7	18.4	2.89	33.0	66.0
37-2	50	311	13	33	179	78	359	12	307	07	14.0	17.5	3.08	34.1	68.2
38	50	039	21	21	219	61	039	29	039	27	14.1	17.7	3.04	33.9	67.8
39	50	019	10	25	259	76	079	14	017	06	13.9	17.4	3.11	34.3	68.5
40	50	223	08	27	217	76	037	14	048	11	11.7	14.6	3.98	37.7	75.4
41	50	228	04	31	180	85	000	05	228	03	14.1	17.7	3.04	33.9	67.7
42	50	328	32	27	192	61	012	29	331	28	15.8	19.8	2.63	31.4	62.8
43	50	358	38	32	187	70	007	20	358	25	12.1	15.1	3.79	37.1	74.2
44	50	348	19	23	184	78	004	12	348	15	11.1	13.8	4.34	38.7	77.4
45	75	302	09	26	137	79	317	11	301	06	11.6	14.5	3.00	50.3	67.1
466															
-1	50	020	00	19	180	77	000	13							
-2	50	036	18	26	284	64	104	26							
-3	50	182	21	33	318	62	138	28							
-4	50	043	12	22	303	62	123	28							
-5	50	087	18	32	281	70	101	20							
-6	50	076	07	29	350	85	170	05							
-7	50	062	20	36	203	50	023	40							
-8	50	178	28	23	010	79	190	11							
-9	50	298	15	28	130	84	310	06							
-10	50	323	05	22	180	75	000	15							
-11	50	036	14	33	272	47	092	43							
-12	50	345	16	21	146	68	326	22							
505	50	000	29	44	091	10	271	80	000	28	17.7	22.2	2.29	28.6	57.1
506	50	243	59	42	302	08	122	82	240	62	17.5	22.0	2.31	28.8	57.5
507	50	065	22	44	357	09	177	81	061	23	18.0	22.6	2.24	28.1	56.2
508	50	011	14	26	220	88	040	02	012	04	15.0	18.8	2.78	32.4	64.8
509	50	211	04	26	180	85	000	05	215	01	14.6	18.3	2.90	33.1	66.2

Evaluation of First Method

The Method

After the statistics for the writer's samples had been calculated and compared with the equal-area projections of the data, it became apparent that the method, even with the improvements described in the previous section, had several flaws. Briefly, these are:

1 The method is incapable of satisfactorily handling multimodal distributions, yet these are the rule rather than the exception for the a-axes of till pebbles, with which the method is intended to deal.

2 The method misinterprets and misuses the precision parameter, k.

3 Watson's (1956b) significance points for R, the length of the resultant vector, are not valid for non-directed orientations.

Comparison of the results obtained using the approach of Andrews and Shimizu (1966) with the point density diagrams for the same data revealed that in many instances the calculated statistics were neither useful nor meaningful in terms of the actual preferred orientations of the till pebbles. This led to a closer examination of the suitability, for the treatment of the a-axis orientations of till pebbles, of the method devised by Watson and Irving (1957) for the treatment of paleomagnetic vectors, and to the clarification of certain important differences between the two applications.

Briefly, the relevant aspects of the differences between magnetic vectors and pebble axes may be stated as follows. (1) While magnetic

vectors are directed, pebble axes are non-directed. This necessitates, as described previously, the selection of a suitable hemisphere before the orientations can be treated as vectors. (2) While paleomagnetic vectors are tightly grouped about a single direction, the a-axes of till pebbles are loosely grouped about (in most cases) two or more directions.

The way in which these differences affect the procedure recommended by Andrews and Shimizu (1966) for the treatment of a-axis orientations will be discussed below, considering in sequence each of the eight steps in their procedure.

"(1) Obtain till fabric data and feed it into the computer program.

"(2) Test the hypothesis that the three-dimensional distribution is random by evaluating R against an acceptable confidence level, e.g., 90, 95 or 99 per cent.

"(3) Accept or reject the null hypothesis; if 'accept' the analysis must stop, if 'reject' the analysis proceeds to (4)."

Watson's (1956b) test for randomness of directions was designed for vectors distributed on the sphere according to Fisher's probability function (Watson, 1956b, p.160). In this situation, non-randomness would appear as a tendency for the observed vectors to cluster toward one side of the sphere, rather than being distributed over the whole sphere.

Pebble axes are non-directed, and, before they can be treated using a vector method, must be "cut in half" to lie in a selected hemisphere. Thus, what is actually dealt with vectorially is a set of vectors distributed over a hemisphere. But a set of vectors lying in a hemisphere constitutes a clustering or preferred orientation when compared to randomness over the whole sphere. Hence any set of vectors in a

hemisphere should yield a significant value of R, indicating non-randomness on the sphere. That is, a set of randomly distributed non-directed orientations would, when sense was assigned by selecting a hemisphere, give rise to a set of vectors distributed randomly over the hemisphere. However, this set of vectors would yield a significant value of R, since it is <u>not</u> randomly distributed over the sphere.

If R is the magnitude of the resultant of a set of N unit vectors distributed randomly on the sphere, R is approximately distributed as

$$\sqrt{\frac{N X_3^2}{3}}$$

where X_3^2 stands for a chi-square variate with three degrees of freedom (Watson, 1956b, p.161). For N=50, the 95 per cent, 99 per cent, and 99.5 per cent points of this distribution are respectively 11.4, 13.7 and 14.6. This means that if 50 unit vectors are selected randomly from a population of unit vectors that is uniformly distributed over the sphere, the probability is only 0.5 per cent or one in 200 that the magnitude of the resultant will exceed 14.6. That this is not true in the case of a distribution in a hemisphere is demonstrated by samples 505, 506 and 507 (fig. 5), each consisting of 50 orientations selected randomly from a population which was uniformly distributed over a hemisphere. The R values for samples 505, 506 and 507 are respectively 28.6, 28.8 and 28.1.

Clearly, the use of this test in the procedure of Andrews and Shimizu (1966) is not valid. In any case, the correct approximation for R is

$$\sqrt{\frac{N X_3^2}{3}} \text{ (Watson, 1956b, p.161)}$$

not

$$\sqrt{N \chi^{2}_{\frac{95}{3}}} \qquad \text{(Andrews and Shimizu, 1966, p.156).}$$

"(4) Plot the values for orientation and dip as well as the limits of the confidence area which has been determined prior to the analysis. This involves rerotation if rotation has already been applied to the observations.

"(5) Ascertain whether the sample approximates a spherical normal distribution, the <u>lower limit of acceptance</u> of this hypothesis being $k \geq 3$. If it does not approximate a spherical normal distribution, then the analysis must cease or an attempt be made to improve the value of k by rotation of the data. Implicit in this <u>lower limit of acceptance</u> is the fact that, with k equal to 3 for 95 per cent of the time, observations will lie within a radius of $\pm$ 90 degrees about the mean vector. As k increases this limit becomes smaller." (Andrews and Shimizu, 1966, p.156)

This step involves an incorrect interpretation of the precision parameter, k. When the distribution is <u>known</u> to be spherical normal (this implies unimodality) k gives information about the shape of the spherical normal distribution, that is, the degree of scattering of the data. k gives no information about the <u>kind</u> of distribution; in fact, if the distribution is not spherical normal, the parameter k, as used by Fisher (1953) and Watson and Irving (1957), is meaningless. The value of k cannot be used to distinguish spherical normal distributions from distributions of other kinds, since a spherical normal distribution may have any value of k from zero to plus infinity.

Hence, $k < 3$ does not imply that the distribution is not spherical

normal, nor can $k > 3$ be used to justify the assumption that the distribution is spherical normal. The assumption of a spherical normal distribution must be justified in terms of the purpose of the entire procedure, and whether the assumption aids or defeats that purpose. This question is discussed fully later.

What Andrews and Shimizu intended in step (5) is revealed by the second two sentences of the step. The significance of the value $k=3$ is that a Fisher distribution with $k > 3$ has more than 95 per cent of the total probability within 90 degrees of the mean direction, that is, more than 95 per cent of the probability lies in one hemisphere. As mentioned previously, a set of non-directed orientations gives rise to a set of vectors that is confined to a hemisphere. If this is to be approximated by a Fisher distribution, it is clearly necessary that the approximating distribution be essentially confined to a hemisphere, that is, it must be essentially zero over the hemisphere farthest from its mean direction. If this condition is deemed to be satisfied by a Fisher distribution having at least 95 per cent of the total probability within one hemisphere, then the minimum permissible value of k may be readily computed. Watson and Irving (1957, p.290) have shown that the probability that a direction will be observed which makes an angle of ϕ_o or more with the true mean direction is, when $k > 3$, given with good accuracy by the formula

$$Pr(\phi > \phi_o) = \exp\left\{-k(1-\cos\phi_o)\right\}$$

If ϕ_o=90 degrees,

$$Pr(\phi > 90 \text{ degrees}) = \exp\left\{-k\right\}$$

Hence, if $Pr(\phi > 90 \text{ degrees}) = 0.05$, then

$$0.05 = \exp\left\{-k\right\}$$

and $k=3$. This result must be regarded as only approximate, since the

formula used is not accurate for $k < 3$.

Thus, Andrews and Shimizu were correct in saying that the procedure must stop if $k < 3$. However, the true reason for this is the one just described, and has nothing to do with the validity of the assumption of a spherical-normal distribution.

> "(6) Examine other samples in the same manner and compare the results by use of the maximum F test (see above). If variance is homogeneous proceed to (7)." (Andrews and Shimizu, 1966, p.156)

In principle this step is valid, assuming the statistics used really represent the samples. However, Andrews and Shimizu incorrectly stated that the precision parameter k "measures homogeneity of variance and can be tested by the maximum F ratio (Hartley, 1950) where the degrees of freedom are 2(N-1) and all samples have an equal number of observations." The parameter k represents the variance of the sample; the ratio of the largest k to the smallest k in a set of samples is the "maximum F ratio", and it is this that measures homogeneity of variance and can be referred to F-tables, not k itself. Furthermore, Hartley (1950, p.312) stated that when estimates of variance are based on different degrees of freedom (that is, the samples have different numbers of observations) the maximum F ratio can still be used as a "rough indicator of heterogeneity" (Hartley, 1950, p.312) if the mean number of degrees of freedom is used and provided the degrees of freedom "do not differ considerably".

> "(7) Use the F test to ascertain if sample orientations belong to the same parent population." (Andrews and Shimizu, 1966, p.156)

This step is also valid in the treatment of a-axis orientations, provided the correct statistic

$$\frac{(N-q)(\Sigma R_i - R)}{(q-1)(N-\Sigma R_i)}$$

where N is the total number of observations, q the number of samples, R_i the length of the resultant of the i-th sample, and R the length of the resultant of all the observations combined (Watson and Williams, 1956, p.348) is used.

"(8) Use the results in an analysis of variance design to determine precision parameters ω and β (Watson and Irving, 1957)."

This is a valid and useful procedure when applied to sets of orientations that have Fisher distributions about their means, the purpose of which is the estimation of the overall mean direction and the prediction of the optimum sampling pattern. However, it must be remembered that "This treatment is approximate, being valied for small dispersions only, that is ω and β large" (Watson and Irving, 1957, p.297), unless the modified procedure for situations involving variable ω is used (Watson and Irving, 1957, p.299). It is not a valid procedure when applied to sets of orientations that do not have Fisher distributions about their means. Andrews and Shimizu did not test the fit of their data to a Fisher distribution. It will be shown below that a model of a single Fisher distribution is invalid for all but a very small proportion of till fabric samples.

The biggest single flaw in Andrews and Shimizu's (1966) approach of subjecting till fabric data to Watson and Irving's (1957) procedure

for the statistical treatment of paleomagnetic vectors is that the observed orientations of pebble a-axes are treated as though they had a Fisher distribution in a selected hemisphere. However, Andrews and Shimizu (1966, p.156, 159), Andrews and King (1968, p.446), Kauranne (1960, p.94), Holmes (1941, p.1312), and Harrison (1957, p.295), using data from such widely scattered areas as northern England, central Finland, and northeastern U.S.A., have commented on the prevalence of distributions with both parallel and transverse maxima. Holmes (1941) further suggested that pebbles of certain shapes tend to align themselves obliquely to the ice movement direction, thus implying the possibility of a-axis distributions having more than two maxima.

The effect of assuming a Fisher distribution when the actual distribution is bimodal can be illustrated by the results obtained using Andrews and Shimizu's (1966) procedure initially applied to the data. For example, sample 1 (fig. 6) shows two maxima at nearly 90 degrees. The most reasonable interpretation of this, in the light of both previous work and the results of the present study, is that the stronger maximum at 352 09 represents pebbles aligned parallel to ice movement and the weaker maximum at 064 25 pebbles transverse to the movement direction. The whole-fabric mean calculated on the assumption of a Fisher distribution lies at 017 09 and clearly does not give a good indication of the ice movement direction.

Sample 5 (fig. 6) is another example of a good parallel/transverse type of distribution. The two maxima lie at 034 00 and 139 08, the first of these being the stronger. The whole-sample mean based on the assumption of a Fisher distribution is 184 03, again failing to indicate the movement direction.

A third good example of this type of distribution is sample 10 (fig. 7). Here the parallel maximum is at 009 09 and the transverse maximum at 104 00. The whole-fabric mean is 041 07, about halfway between the two maxima.

In some samples of this type, for example, sample 7 (fig. 6), the whole-fabric mean does lie close to the parallel maximum. However, this occurs only when the proportion of transverse pebbles is quite small and the distribution has almost perfect symmetry about the plane passing through the parallel maximum and normal to the transverse maximum.

One of the aims of treating till fabric data by the vector summation method is to determine the ice movement direction as indicated by the strongest preferred orientation of the long axes of the till pebbles. The above results show that the effect of transverse pebbles, that is, those lying in a weaker preferred orientation at 90 degrees to the ice movement direction, is to distort the calculated mean away from the direction of maximum density of observations so that it lies somewhere between the two preferred directions. Thus, if it is required that the calculated mean coincide with the stronger preferred direction, (that is, the parallel maximum), then pebbles lying in the weaker preferred orientation (that is, the transverse maximum) must be deleted before the mean is calculated.

Andrews and Shimizu (1966, p.159-160) mentioned the problem of transverse pebbles, and proposed the solution "that 10 per cent of the observations lying $\pm$ 90 degrees from the preferred orientation after a 90-degree rotation has been completed, be omitted from the calculations and the results rerun." They reran their samples using this procedure,

obtaining increased values of k and R/N, and concluded that "Clearly, all statistics increase in significance."

As nearly as can be determined from the statement of the procedure quoted above, and considering the computer program used by Andrews and Shimizu, it appears likely that those pebbles were deleted whose azimuths lay closest to the directions 90 degrees from the azimuth of the calculated mean vector. This procedure has two minor drawbacks: (1) the actual proportion of transverse pebbles in tills varies greatly from the assumed 10 per cent, and (2) if the mean vector is not horizontal, deleting the pebbles on the basis of a 90-degree difference of azimuth does not delete those pebbles that are actually 90 degrees from the mean vector. This error becomes more serious as the plunge of the mean vector increases. The procedure also has one major drawback: if there are transverse pebbles, as is usually the case, the whole-sample mean direction will frequently not coincide with either maximum of the distribution, as described and illustrated above, and hence the transverse pebbles do not lie at 90 degrees to the calculated mean direction. In short, deletion of transverse pebbles cannot be carried out until the transverse direction has been located, but the directions of the fabric maxima cannot be located by Andrews and Shimizu's procedure until transverse pebbles have been deleted. The statistics k and L must increase in value if pebbles lying close to 90 degrees from the calculated mean are deleted and the data rerun; this does not imply that the new mean is any closer to the stronger maximum of the distribution.

Andrews and Shimizu suggested that $k < 3$ indicates the presence of enough transverse pebbles to be a nuisance and warrant deletion. This is revealed by their step (5) in which the value of k is taken to indicate

how well the data fit a Fisher distribution, and by the following statement in their "Summary and Conclusion": "in some samples there is a large number of transverse pebbles which decrease the precision parameter to <3." (Andrews and Shimizu, 1966, p. 161).

Several of the samples used in the present study yielded a value of $k < 3$ on initial processing using Andrews and Shimizu's procedure. These samples were rerun with all observations lying more than 60 degrees from the calculated mean deleted. Although an arbitrary angular cutoff was used instead of an arbitrary percentage cutoff, the procedure is basically the same as that used by Andrews and Shimizu, except that the true deviation angles were used instead of azimuth differences, thus making the procedure more precise. The results obtained before and after deletion are listed in Table 3.

In every case, θ_{95} (the 95 per cent confidence radius on the position of the mean) decreases and both k and L(=R/N) increase. However, it is interesting to examine the effect of the deletion on the positions of the means. In all cases, the means move by only a few degrees, so that if the mean fell near a maximum before deletion, it remained near the maximum after deletion, and if the mean was not close to a maximum before deletion it was not close to one after deletion. In other words, the only significant effect of the deletion was to increase the statistics k and L and decrease θ_{95}.

Andrews and Shimizu appear to have been satisfied by this increase in numerical value of the parameters k and R/N. However, the reason for deleting transverse pebbles is to eliminate their effect of distorting the calculated mean away from the region of maximum density of observations. Andrews and Shimizu failed to ascertain whether or not

	Whole Sample				Observations within 60 degrees of whole sample mean			
Sample number	Mean	95% Conf. Rad. (deg.)	k	L=R/N (%)	Mean	95% Conf. Rad. (deg)	k	L= R/N(%)
3	049 15	14.1	2.65	62.8	047 18	11.9	4.44	78.0
4	304 05	14.9	2.84	65.5	296 00	12.3	4.63	79.0
5	184 03	14.3	2.91	66.3	181 02	12.5	4.35	77.6
17	277 18	14.6	2.90	66.3	280 12	12.1	4.76	79.6
20	323 05	14.4	2.97	67.0	316 05	11.4	5.35	81.8
23	027 00	16.0	2.58	62.1	214 04	14.9	3.36	71.0
25	000 14	15.9	2.62	62.6	001 20	12.9	4.73	79.5
29	046 05	15.5	2.70	63.7	043 05	12.7	4.66	79.1
31	294 14	16.8	2.45	59.9	297 18	14.5	4.18	76.8
32	141 01	15.2	2.75	64.4	146 06	12.5	4.43	78.0
33	332 27	14.7	2.87	65.9	330 23	11.4	5.24	81.4

Table 3. Changes in k and L on deleting "transverse" pebbles by the procedure recommended by Andrews and Shimizu (1966). k = estimate of K, the precision parameter of the population; L = vector magnitude = R/N.

this objective had been achieved; the results described above and listed in Table 3 indicate that for the data used in the present study (apparently not abnormal for till fabric data) Andrews and Shimizu's method of deleting "transverse" pebbles is unsuccessful.

Thus, if Andrews and Shimizu's recommended procedure is followed for the nearly-ideal parallel/transverse patterns of samples 1, 5, 7 and 10 (figs. 6, 7), the unrealistic mean of 017 09 for sample 1 and the unrealistic mean of 041 07 for sample 10 will both be accepted since in both cases $k > 3$. The unrealistic mean of 184 03 will be rejected for sample 10 since $k < 3$, but after "transverse" pebbles have been deleted, $k > 3$ and the still-unrealistic mean of 181 02 will be accepted. The realistic mean of 046 06 for sample 7 will be accepted since $k > 3$.

These samples represent nearly ideal simple fabrics, yet in three cases out of four the mean calculated by Andrews and Shimizu's method fails to indicate the ice movement direction or have any other useful meaning.

Table 4 shows a number of hypothetical a-axis distributions of till pebbles, their possible appearance in a sample, the useful information that can be obtained from the sample, and whether or not Andrews and Shimizu's method of representing the sample can be expected to reveal this information. The only cases for which the method is useful are those in which all maxima of the distribution are parallel maxima produced by ice movement and only the mean direction of movement is required. In all other cases some other method of representing the distribution must be used.

Hypothetical Fabric Pattern	Sample	Information Required from Sample	Method Works
Single parallel mode or single transverse mode	1 mode	Position and strength of mode	Yes
Parallel and transverse modes	2 modes @ 90°	Positions and strengths of modes. Mean not acceptable.	No
Parallel and transverse modes	2 modes not @ 90°	"	No
Parallel and oblique modes		"	No
Transverse and oblique modes		"	No
Modes due to ice movement and later shearing		"	No
Modes due to ice movement and later gravity movement		"	No
Two parallel ice-movement modes		" Mean may be acceptable	Maybe
Parallel, transverse and oblique modes	More than 2 modes	" Mean not acceptable	No
Several parallel ice-movement modes		" Mean may be acceptable	Maybe

Table 4 Hypothetical fabric patterns, the possible appearance of samples from these fabrics, and an indication of whether or not Andrews and Shimizu's (1966) method reveals the information required from the sample.

Applications of the Method

Andrews and King (1968) employed the method of Andrews and Shimizu (1966) in studying till fabric variability in northern England. They carried the method through to the stage of calculating the within- and between-site variability parameters ω and β to determine the optimum sampling pattern.

The fallacious tests based on R and k are still used; however, the method of selecting an axis for rotating the data was an improvement over the fixed east-west axis used by Andrews and Shimizu (1966). A 90-degree rotation was used, "with the axis of rotation passing through the area of minimum observations" (Andrews and King, 1968, p.444). This is a better attempt to approximate the "minimum variance" position of the reference hemisphere.

Because of the problems already discussed, Andrews and King's calculation of ω and β and the estimation of the optimum sampling pattern cannot be regarded as valid. Andrews and King's claim that their calculated optimum sampling pattern is "surprising but is statistically sound" (Andrews and King, 1968, p.457) is itself surprising in view of their own statement that the calculated means and k values do not represent Fisher distributions or even unimodal distributions (Andrews and King, 1968, p.446).

In addition to using the method of Andrews and Shimizu (1966) (with a fixed axis of rotation), Cowan (1968) determined the directions of fabric maxima using a X^2 test on a plot of a-axis trends, as well as plotting point diagrams on polar projections. No fabric diagrams were

presented in the paper, however; only the statistics calculated using X^2 on the trend distribution and Andrews and Shimizu's (1966) method on the three-dimensional distribution. Although Cowan was thus able to check the mean directions given by Andrews and Shimizu's (1966) procedure against graphical representations of the data, he was misled into paying unwarranted attention to the mean directions given by Andrews and Shimizu's method by the supposedly high degrees of significance of the parameter R: "Orientation and vector strengths indicate that none of the fabrics were random and therefore some explanation should be attributed to the mean orientations derived from these." (Cowan, 1968, p.1154).

Cowan was misled also by Andrews and Shimizu's (1966, p.156) incorrect statement concerning k: "This statistic k, measures <u>homogeneity of variance</u> and can be tested by the maximum F ratio." Led by Andrews and Shimizu's use of the condition $k \geq 3$ (Andrews and Shimizu, 1966, p.156) into believing that values of $k > 3$ had some kind of statistical significance, Cowan concluded "k values (where $k = (N - 1)/(N - R)$) for the homogeneity of variance are significant for both proximal and distal fabrics in this example. This means that they may be tested further, if necessary, to determine whether or not the mean directions of populations are identical" (Cowan, 1968, p.1150). It is the ratio of k values that tests homogeneity of variance, not individual k values.

Since the numerical procedure for representing till fabric data used by Cowan (1968) is known to be of little value in many cases, the reader is left without any reliable indication of what the observed distributions were actually like. This situation emphasizes the necessity of presenting till fabric data graphically, at least until a numerical procedure has been devised and widely accepted that is valid in all

cases and adequately describes the distribution.

Caine (1968) must be given credit for recognizing the inability of Andrews and Shimizu's (1966) method to locate the preferred orientations when significant numbers of transverse pebbles are present, and for apparently attaching no significance to the calculated value of R. In applying the method to the orientation of periglacial blockfield material of Tasmania, Caine (1968) used stereographic projections and the results of two-dimensional analysis to determine the preferred orientations of the blockfield particles. He then applied Andrews and Shimizu's (1966) procedure separately to the observations lying in the two preferred orientations (parallel and transverse to the local slope direction), separating the two by a constant 30 degree/60 degree cutoff, after rotating the data 90 degrees about a line normal to the slope direction.

Caine cited the results of the three-dimensional analysis as tending to corroborate other evidence that the blockfield material was preferentially oriented parallel to the local slope direction. However, suppose Caine's procedure is followed using a set of random orientations. The data are rotated 90 degrees about a horizontal line perpendicular to the local slope direction, and the mean of the orientations <u>then</u> having trends $\pm$ 60 degrees from the local slope direction calculated. This mean will lie close to the vertical. When it is rotated with the data back to the original position, it is very likely that the final position of the mean will be close to the local slope direction; the procedure followed ensures that this will be so.

Second Method Examined

The first method sets up a single Fisher model for a whole till fabric sample. This approach fails because most till fabrics are multimodal. It is suggested that if the Fisher model is to be used to describe till fabric samples, the modes must first be located and a Fisher model then constructed for each separate mode. An attempt to design such a procedure is described below.

A procedure for numerically representing a distribution of orientations must be based on (1) the nature of the distribution and (2) the information required. For the long axes of till pebbles the relevant considerations are: (1) The long axes of till pebbles characteristically exhibit a high degree of variability in their three-dimensional distribution pattern, even over relatively short distances within a single till unit. This variability is evident not only in the preferred orientations of the long axes but also in the number of preferred orientations and the angles between them. (2) As indicated in Table 4, in most cases the information required is given by the positions and the relative strengths of the modes of the distribution.

Thus, a procedure for treating the data should (1) locate the important modes of the distribution, (2) determine the relative strengths of the modes, (3) describe the modes numerically in a way that permits statistical comparisons with other samples. A procedure for statistical representation and comparison of samples must be based on a model that is at least approximately valid for all samples tested. It has been demonstrated in foregoing sections that a model of a single Fisher distribution for a sample of long-axis orientations of till pebbles is

rarely valid and a procedure based on this model does not give the information required. Any unimodal model would be similarly invalid.

None of the methods presently available for calculating descriptive statistics for orientation data is directly applicable to bimodal or multimodal distributions. Even if the methods were so applicable, they could not be applied until each sample had been appraised visually to determine which model was valid for that sample, since no statistical methods are available for testing a three-dimensional distribution of orientations against models any more complex than a single Fisher distribution.

Thus none of the numerical procedures available is suitable for treating directly a sample of long axis orientations of till pebbles.

It is suggested that bimodal and multimodal distributions can be treated by a method based on a unimodal model if the observed distribution is first subdivided into a number of separate unimodal distributions. Such an approach formed the basis of Andrews and Shimizu's (1966) suggested technique of separating the modes of the distribution by vertical planes using a two-dimensional plot of the trend distribution. This would be adequate for the purposes of a two-dimensional treatment, but it is felt that when a three-dimensional treatment is being used, a method of subdividing the observed distribution based on its three-dimensional characteristics is far more suitable. A procedure will be described whereby this can be accomplished easily and effectively.

The basis of the suggested approach is the hypothesis that the observed bimodal or multimodal distribution can be approximated by two

or more superimposed Fisher distributions, each having a different mean position, different K, and different relative magnitude. A generalized procedure for dealing with such a distribution will first be described. The validity of the model for till fabric samples will be discussed in the following section, "Evaluation of Second Method".

The first stage of the suggested procedure is the production of a contoured point density diagram. This can be accomplished using the computer program described in Chapter 2 and given in Appendix D. The second stage involves examination of the point density diagram and the selection of a model for the sample. This step is discussed fully in Chapter 4. Basically the procedure is one of comparison with various models, and selection of the simplest model that is consistent with the sample. The selected model will have zero, one or two modes if the sample size is 50 (50 measurements are not adequate to indicate reliably more than two preferred directions). Using a stereonet, the trend and plunge of the approximate centre of each mode can be read off, together with an angular radius defining the range of the mode on the surface of the reference sphere. These values thus define a circle on the sphere (or a cone whose apex lies at the origin of the reference axes) that contains the observations that define the mode. Such a circular or conical region is thought to be the most realistic way of delimiting each mode, since the model to be used for the mode is a Fisher distribution, which has axial symmetry. Further, such a delimitation is very suitable for use in the computer program that constitutes the next stage in the procedure.

The mean vector program described earlier with a slight modification, is now used. As previously described, this program calculated the

position of the mean and other statistics on a set of orientations using the specified hemisphere. The hemisphere was specified by the direction of its central axis. This program was modified by adding an angular limit, θ ($\leq 90^{\circ}$). The modified program now calculates the same statistics, but uses only those observations making an angle not greater than θ with the specified axis. Thus it is now possible to represent those observations lying within a cone with its axis in any desired position and with any desired semi-angle, θ (that is, observations lying within a circle of any radius ≤ 90 degrees in any position on the sphere). The number of observations lying in the specified region is also recorded, thus providing an indication of the relative strengths of the modes. After these statistics have been calculated for each mode, they may be used in the relevant tests described by Watson and Irving (1957), provided the model can be shown to be valid. Thus all the requirements stated above have been satisfied: the modes have been located (graphically), an indication of the relative strengths of the modes has been obtained, and statistics calculated for comparison with other samples.

Evaluation of Second Method

The validity of comparisons of samples using the statistics calculated by the method just described depends upon the validity of the assumption that the observations selected by the specified cone actually have a Fisher distribution about their mean. So that this could be tested, the mean vector program was further modified to incorporate a test of fit of the data to a Fisher distribution with the estimated parameters. Details of this modification and the test of

fit may be found in Appendix D.

Tests were run on 78 selected modes on 42 different diagrams. The cones used and the results obtained are listed in Table 5. The first point to be observed about these results is that 40 of the 78 cones contained less than 20 observations, too few to calculate a chi-square value for either the polar or axial distribution. A further 20 contained more than 20 observations (permitting calculation of a chi-square value for the axial distribution) but still too few for calculation of a value for the polar distribution. Hence only 18 of the 78 cones contained sufficient observations to permit calculation of chi-square parameters for both polar and axial distributions.

If the hypothesis being tested is rejected when a chi-square value is obtained that exceeds the value at the 95 per cent point, every one of the 18 sets of observations that could be tested would be rejected on at least one count (that is, either the polar test or the axial test or both). If the 99 per cent point is used as the criterion of rejection, 13 of the 18 would still be rejected on at least one count.

Two facts become evident from these results: (1) For this method to be useful, much larger samples would be necessary than were used in the present study. (2) Few of the modes tested even approximate a Fisher distribution. This implies that statistical tests using the sample means and k values calculated are not valid. In view of this, no such tests were attempted.

TABLE 5. Statistics calculated using second method and results of test of fit to a Fisher distribution. Column headed "Pr%" gives the probability of obtaining the associated value of X^2 under the hypothesis being tested. Four points were used: 5%, 2½%, 1%, and ½%. Each probability is stated as "less than" the smallest of these four probabilities that exceeds it. Probabilities greater than 5% are so indicated. All X^2 values have one degree of freedom. A = semi-angle of cone. n = number of observations within cone. CS = chi-square.

STA NO.	CONE AXIS TR	CONE AXIS PL	A	n	RESULTANT TR	RESULTANT PL	RESULTANT R	CONF. RAD. 95%	CONF. RAD. 99%	K	L(%)	TEST OF FIT POLAR CS	TEST OF FIT POLAR PR%	TEST OF FIT AXIAL CS	TEST OF FIT AXIAL PR%
1	352	09	45	31	354	06	28.3	8.1	10.1	11.25	91.4	2.3	>5	5.3	<2½
2	328	18	25	8											
2	029	20	35	23	028	18	22.0	6.5	8.2	22.52	95.8			5.8	<2½
3	072	27	40	22	069	23	20.4	8.9	11.3	13.06	92.7			3.6	>5
3	003	06	40	25	003	08	22.9	9.1	11.4	11.21	91.4			5.8	<2½
4	093	21	45	25	093	16	22.7	9.4	11.9	10.40	90.8			4.5	<5
5	034	00	35	17	213	01	16.1	8.6	10.9	18.34	94.9				
5	139	08	30	16	138	05	15.2	9.0	11.5	17.66	94.7				
6	072	02	70	44	072	03	35.5	10.7	13.4	5.05	80.7	8.4	<½	11.5	<½
7	308	04	22	6											
7	039	07	70	45	038	06	38.6	8.7	11.0	6.90	85.8	7.1	<1	45.9	<½
8	323	45	80	47	329	46	36.7	11.2	14.0	4.47	78.1	4.8	<5	14.8	<½
8	099	30	15	2											
9	325	14	25	7											
9	039	10	50	39	041	09	35.3	7.5	9.4	10.36	90.6	7.8	<1	8.6	<½
10	009	09	40	25	010	08	23.8	6.7	8.4	19.73	95.1			4.2	<5
10	284	00	35	16	102	01	15.3	8.3	10.5	20.88	95.5				
11	224	14	25	14	225	13	13.3	9.2	11.9	19.44	95.2				
11	330	13	30	7											
11	272	02	35	18	270	04	17.1	7.9	10.1	19.92	95.3				
12	099	14	30	7											
12	020	03	50	37	203	02	33.3	8.0	10.0	9.67	89.9	4.7	<5	8.3	<½
13	025	00	35	14	027	00	13.3	9.7	12.4	17.77	94.8				
13	137	08	45	36	136	05	32.9	7.5	9.4	11.21	91.3	2.7	>5	6.9	<1
17	269	16	25	12	270	17	11.5	9.1	11.8	23.55	96.1				
17	120	06	25	10	122	08	9.7	8.7	11.3	31.95	97.2				
18	330	31	70	47	328	28	39.4	9.3	11.6	6.02	83.8	10.2	<½	26.5	<½
20	302	03	60	35	305	05	29.4	10.8	13.5	6.04	83.9	2.8	>5	4.9	<5
20	028	00	35	12	025	00	11.3	11.6	14.9	15.00	93.9				
21	041	12	50	30	039	10	26.2	10.1	12.8	7.71	87.5	5.3	<2½	6.6	<2½
21	057	07	30	14	058	09	13.4	9.0	11.5	20.56	95.5				
21	014	13	25	11	016	14	10.6	8.8	11.4	27.91	96.7				
22	328	40	20	5											
22	284	20	35	17	281	16	15.8	10.2	13.0	13.21	92.9				
22	070	20	45	32	071	12	28.7	8.8	11.0	9.35	89.6	7.2	<1	3.3	>5
23	244	28	35	16	246	29	15.2	8.8	11.2	18.54	94.9				
23	148	20	25	6											
24	352	26	30	20	357	25	19.2	7.0	8.9	22.76	95.8			14.8	<½
24	042	16	30	21	039	17	20.1	6.8	8.6	23.12	95.9			9.0	<½
25	208	02	25	9											
25	341	40	35	18	339	40	16.9	9.0	11.5	15.71	94.0				
26	341	11	45	37	339	14	33.8	7.3	9.2	11.28	91.4	2.0	>5	8.1	<½
27	025	00	45	29	206	02	26.1	9.0	11.3	9.79	90.1	5.6	<2½	16.1	<½
28	287	27	20	3											
28	048	36	30	8											
28	000	02	40	25	357	01	23.4	7.8	9.9	14.66	93.5			5.5	<2½
29	100	02	40	19	097	02	17.4	10.5	13.3	11.18	91.5				
29	028	08	40	22	022	07	20.2	9.6	12.2	11.37	91.6			6.8	<1

TABLE 5 (continued)

STA	CONE AXIS				RESULTANT			CONF. RAD.				TEST OF FIT POLAR		AXIAL	
NO.	TR	PL	A	n	TR	PL	R	95%	99%	K	L(%)	CS	PR%	CS	PR%
30	165	43	40	21	160	44	19.3	9.5	12.1	12.10	92.1			3.0	>5
30	105	16	25	9											
31	050	00	25	10	229	01	9.6	10.0	13.0	24.53	96.3				
31	350	18	30	12	346	18	11.6	8.9	11.5	24.69	96.3				
32	012	20	30	11	009	23	10.5	10.1	13.0	21.53	95.8				
32	135	12	40	22	132	13	20.1	9.8	12.4	10.95	91.3			2.8	>5
33	289	29	20	9											
33	339	19	40	27	341	17	24.1	9.9	12.5	8.87	89.1			5.1	<2½
34	024	10	26	29	024	09	28.1	4.8	6.0	32.31	97.0	4.3	<5	3.3	>5
34-3	024	11	27	18	025	07	17.3	7.1	9.0	24.83	96.2				
34-4	025	10	26	12	025	11	11.7	6.9	8.9	40.93	97.8				
36	002	20	30	23	000	21	22.0	6.8	8.5	21.01	95.4			9.4	<½
37	313	18	40	41	310	14	37.3	7.1	8.9	10.95	91.1	10.1	<½	8.5	<½
37	204	12	25	16	206	14	15.6	5.7	7.3	42.63	97.8				
37	066	19	30	21	066	18	20.0	7.3	9.2	20.12	95.3			11.8	<½
37-3	320	18	40	21	317	20	18.8	11.1	14.1	9.14	89.6			6.6	<2½
37-3	202	10	26	7											
37-3	074	15	30	9											
37-4	308	06	40	22	307	09	20.2	9.4	11.9	11.78	91.9			4.4	<2½
37-4	207	13	23	10	207	15	9.8	7.8	10.1	39.78	97.7				
37-4	056	20	23	8											
38	076	24	45	27	080	26	24.4	9.3	11.7	9.88	90.3	3.3	>5	5.8	<2½
38	338	14	45	20	342	16	18.2	10.4	13.2	10.80	91.2			6.0	<2½
39	331	00	35	19	150	01	17.9	8.7	11.0	15.92	94.0				
39	041	10	35	20	040	09	18.4	9.7	12.3	12.21	92.2			6.0	<2½
40	045	11	35	30	042	12	28.1	7.0	8.8	15.16	93.6	15.3	<½	8.3	<½
41	109	00	45	18	103	01	15.9	12.9	16.4	8.16	88.4				
41	019	00	45	27	024	01	24.0	10.0	12.6	8.76	89.0			11.8	<½
44	351	14	45	36	350	13	32.9	7.5	9.4	11.21	91.3	1.6	>5	6.9	<1
45	296	07	30	28	295	08	26.3	7.1	9.0	15.66	93.8			6.6	<2½

Summary and Conclusions

Andrews and Shimizu's (1966) suggestion that the treatment of till fabric data could benefit from use of a three-dimensional statistical procedure based on work by Fisher (1953) and others concerning probability distributions on a sphere was a good suggestion. They recognized the problem posed by the bimodal nature of most till pebble a-axis distributions, and proposed the solution of deleting transverse pebbles before treating the data.

The procedure they recommended, however, is not suitable for treating a-axis orientations of till pebbles for the following reasons:

1 The condition $k \geq 3$ is used as the sole criterion of applicability of the method, that is, as indicating that the sample is essentially unimodal. This is not true.

2 The recommended procedure for deleting transverse pebbles fails.

3 Watson's (1956b) test for randomness of directions based on the length of the resultant vector is not valid for hemispherical distributions.

At least one subsequent publication depended heavily on the statistics obtained by Andrews and Shimizu's recommended procedure for representation of till fabric data. Since it has been shown that such statistics are quite unreliable and in many cases physically meaningless, the actual nature of the data used in that study was not communicated. Furthermore, since the conclusions of the study were in large part based on the statistics, those conclusions must be regarded as questionable.

It is important to recognize that the method fails not because it is a bad or unsound method but because it is used in a situation for

which it was not designed. Any statistical procedure is based on certain assumptions about the data being treated; if these assumptions are not valid then the method is not sound. In this case the underlying assumption is that the data conform to a Fisher distribution. Hence Andrews and King's application of the procedure to bimodal distributions (Andrews and King, 1968, p.446) is not valid, contrary to their assertion that "This result may be surprising but is statistically sound." (Andrews and King, 1968, p.457). The importance of constructing valid models for statistical studies of geological orientation data was emphasized by Pincus as early as 1953.

An alternative approach is suggested based on the assumption that a multimodal axis distribution can be adequately described by a model consisting of several Fisher distributions, one corresponding to each mode. A graphical procedure has been described for setting up such a model for a sample of till stone long axes. Testing of till fabric data, however, has shown that the assumption that a Fisher model can adequately describe even a single mode of a till fabric sample is not justified. This may be due to the elongate nature of most till fabric modes, resulting from the preference for a near-horizontal plane observed in the majority of samples. Thus, statistical comparison of samples using the parameters calculated by this method cannot be regarded as rigorous.

The calculation of descriptive statistics for till fabric samples based on valid three-dimensional models must await further work on either (1) models for elongate groupings or (2) procedures permitting a grouping to be treated separately from a girdle occurring in the same sample.

Until adequate techniques for the numerical representation of till

fabrics are devised and widely accepted, till fabric data should be presented graphically. This ensures that a publication will not become entirely useless should the numerical procedures used in it be shown to be invalid. Computer programs are now available that reduce drastically the man-hours involved in the production of stereographic diagrams, as well as eliminating inaccuracy and subjectivity.

Possibilities for Future Work

Work that needs to be done on the numerical treatment of distributions of long axes of till stones may be divided into two kinds: (1) new procedures based on presently available models, and (2) establishment of new models.

New Procedures with Present Models

Watson (1956b) derived the distribution of R, the length of the resultant vector, for a set of unit vectors distributed randomly on the sphere, and determined some of the percentage points of this distribution. For the treatment of non-directed axes it would be very useful to know the distribution of R for a set of unit vectors distributed randomly on a hemisphere, and some upper and lower percentage points of this distribution. Observed values of R (or vector magnitude, R/N) could then be compared with the known percentage points and be assessed as significant or non-significant at a suitable confidence level. Such an "R-test" for a hemispherical distribution would actually be more versatile than the same test applied to a spherical distribution. On the hemisphere,

high R values would indicate a tendency for the observations to group about one direction, and low R values would indicate a tendency for the observations to avoid a certain direction or be concentrated into the the plane normal to that direction. Random distributions would have intermediate R values. In the case of a spherical distribution, high R values indicate a tendency for the observations to prefer one side of the sphere; low R values, however, do not indicate deviation from randomness, since low R values are expected for random distributions on the sphere. Since low R values are also expected from distributions having a preferred plane, the "R-test" on the sphere is not suitable for detecting this form of non-randomness.

In the treatment of paleomagnetic vectors a preferred direction is the only form of non-randomness that is of interest; hence R provides a convenient test. In the case of till stone long axes, however, a preferred plane is common and must be considered as a relevant alternative to randomness. Thus the distribution of R for random sets of vectors on a hemisphere would be particularly valuable for till fabric work since (1) pebble axes, being non-directed, must be treated in a hemisphere if treated vectorially, and (2) the value of R in a hemispherical distribution is useful for detecting non-randomness of the forms most frequently found in till fabrics.

Since the hemispherical distribution of a set of non-directed axes depends on the position of the hemisphere, a suitable procedure for using the value of R as an indicator of non-randomness would be as follows: (1) Using many different hemispheres, calculate many values of R. (2) Determine the maximum and minimum values of R. (3) Compare these with known significance points of R.

The vector magnitude program already used performs step (1) (printing R as a percentage of N, the sample size), and permits a rapid graphical solution to step (2). Thus if significance points of R (and therefore of R/N) were known, the vector magnitude diagrams already produced and described elsewhere in this report could become a rapid and simple means of evaluating the probability of non-randomness of the two forms most common in till fabrics -- a preferred direction and a preferred plane.

A complete procedure for evaluating a till fabric sample might be based on this method as illustrated in Figure 8. The existence of a preferred plane and/or a preferred direction would be established by comparing the maximum or minimum value of R determined from the diagram with a suitable percentage point of R. It may be that a strongly preferred plane containing a random distribution of azimuths could contain random groupings that would produce a significantly high value of R. (In other words, a significantly high R value would indicate non-randomness, but not the <u>kind</u> of non-randomness.) For this reason an indication of a preferred plane (that is, a low R value) should be investigated before a significantly high R value is interpreted as indicating a preferred direction. Only if there is no indication of a preferred plane can a high R value be taken directly as indicating a preferred direction.

If no significantly low R value is found, then a significantly high value is looked for (fig. 8). If none is found, the distribution is regarded as random. If one is found, it is concluded that there is a preferred direction and the distribution may be represented by a single Fisher distribution. The appropriate parameters are calculated. (Only

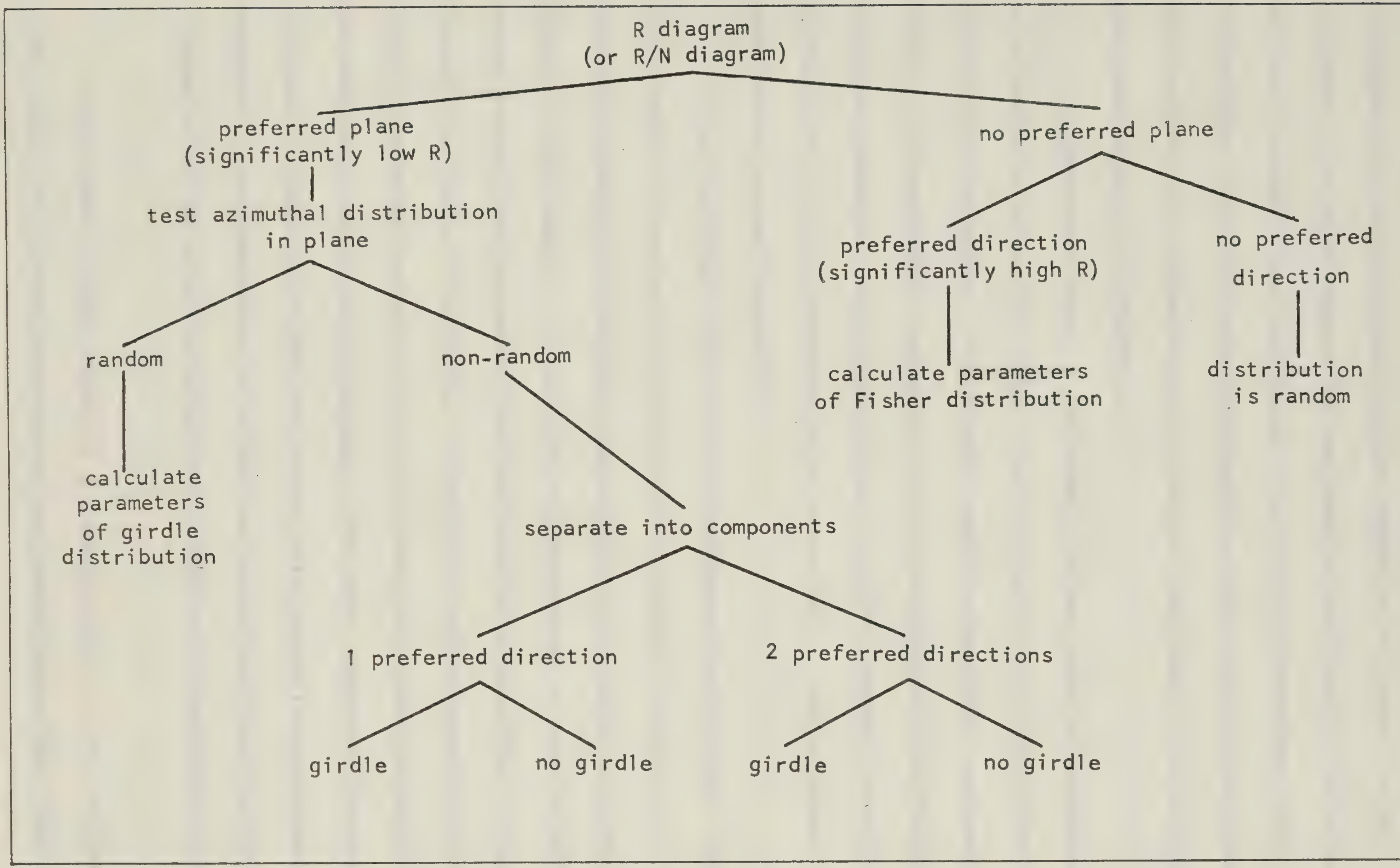


Figure 8 Possible procedure for analyzing a fabric sample based on known significance points of R, the length of the resultant of a set of unit vectors distributed randomly on a hemisphere.

R diagrams with a single peak are considered at this point in the procedure. If the R diagram has two peaks a significantly low value of R will probably have been found, since there is therefore a preferred plane. Three or more preferred directions that are far from being coplanar are not regarded as an alternative that needs to be considered when dealing with till fabrics.)

If a preferred plane is indicated, the next step is to examine the distribution of azimuths in that plane and look for any significant deviation from randomness. A procedure should be used that overcomes the difficulty presented by the possibility of the observed distribution being non-random but having two-fold or four-fold symmetry in the preferred plane. Such a procedure might be similar to that employed in the mean vector program (Appendix D).

If no significant deviation from randomness can be found, the distribution is regarded as consisting of a uniform girdle, and, if desired, parameters of the distribution may be calculated using one of Selby's (1964) models. If the azimuth distribution is found to differ significantly from randomness, the next step is to determine the preferred direction or directions. Ultimately it may be possible at this stage to separate the pattern into its linear and planar components mathematically and calculate parameters for each component (see below). At present, however, one must be content with subjective assessment based on visual examination of either the three-dimensional density diagram or a diagram of the azimuth distribution in the preferred plane.

Both Fisher's (1953) model of a grouping about a line and Selby's (1964) model of a girdle distribution are useful models when they are

applicable. However, one of the main problems of dealing with till fabrics is that the pattern is not only complex but highly variable. If a fabric pattern is considered as being composed of a combination of planar and linear components (girdles and preferred directions), till fabric patterns commonly have a girdle and two preferred directions, a girdle and one preferred direction, or a girdle with no preferred direction. Samples are considered in this study that have no girdle but only a preferred direction (for example, samples 22 and 26 (fig. 9) and 466-2 and 466-4 (fig. 10)). Fisher's model is thus applicable in these latter cases, but for the majority of till fabric samples neither Fisher's nor Selby's distribution alone is adequate to describe the sample. Since Fisher's and Selby's distributions describe the two basic components of till fabric patterns -- the girdle and the grouping about a line -- one is led to hope that it will one day be possible, probably using a computer, to fit a complex model consisting of a combination of the two basic forms to an observed set of orientations. In the meantime, the first step would be to devise procedures (again taking advantage of the computer) to simulate random samples of limited size from hypothetical populations that fit the basic models. This would perhaps lead to procedures for evaluating bimodal and multimodal samples to establish whether two adjacent modes in a sample should be regarded as representing one or two modes in the population. At present this is a matter of guesswork. Simulation could then be carried out using combinations of the basic models, so that a reasonable complex model could be constructed to fit a complex observed distribution, using repeated approximations and visual comparison.

One advantage of such a composite model would be that a single

component of the fabric could be described separately, and compared separately with single components from other samples, without its properties being masked or distrorted by the other components of the fabric. Relationships between different components of fabric patterns could then be studied, since each component could be treated independently.

Improved Models

Many of the fabric patterns considered in this study consist of one or two near-horizontal groupings that are more or less elongated horizontally. This elongation is sufficient in most cases to constitute a significant deviation from a Fisher distribution, thus invalidating statistical procedures based on the assumption of a Fisher distribution. It was suggested above that these elongate groupings might be represented by a model consisting of a Fisher distribution superimposed on a uniform girdle. The compound nature of such a model might, however, make the problem of estimating appropriate parameters very awkward. An alternative approach might be to use as a model a spherical normal distribution <u>without</u> axial symmetry (Pincus, 1953, p.504). If statistical comparisons could be set up on the basis of such a model similar to those presently in use based on a Fisher distribution, they could probably be applied to many till fabric samples that cannot be represented by a Fisher distribution or a combination of Fisher distributions.

Test of Fit Taking Measurement Errors into Account

Since the recorded orientation of an axis is not necessarily the true orientation of the axis, showing that recorded orientations do not have a Fisher distribution does not imply that the true orientations do not have a Fisher distribution. Ideally, a test of fit should be devised that allows for the probable deviations of the observed orientations from the true orientations.

THE PROBLEM OF INTERPRETING PEBBLE FABRIC PATTERNS

Reliability of Samples

Fabric at Sample Site

Introduction

The problem in interpreting a fabric diagram is: What can be inferred about the fabric represented by the sample? This involves separating the random from the non-random elements in the sample, that is, characteristics peculiar to the particular sample from those of the population, in order to establish the presence or absence of a preferred orientation in the population and to determine the preferred orientation if there is one. There are two possible ways of tackling this problem: (1) Comparison of the sample with various models. This is done by setting up, either physically or mathematically, artificial populations with known distributions, then producing random samples from these artificial populations and comparing them with the real sample being evaluated. (2) Comparison of successive samples from the population being studied.

Comparison with Models

One valuable aid is the simulation of random samples from model distributions of various kinds. This gives an idea of the degree to which the form of the distribution is distorted by the random element of the sample for various sample sizes, and thus provides a basis for interpreting a given sample. The following models are useful in dealing with a-axis orientations of till pebbles:

1 Axes uniformly distributed.

2 Axes concentrated in or near a plane, with all parts of the plane occupied with equal frequency (uniform girdle).

3 Axes concentrated in or near a plane, and in that plane near one particular orientation.

4 Axes concentrated in or near a plane, and in that plane near each of two orientations at right angles.

5 Axes concentrated around one orientation, with no plane preferred.

A type-1 distribution and a crude model of a type-2 distribution may be easily simulated. Distributions of types 3, 4 and 5, however, are difficult to simulate, and are best tested for by comparison of successive samples. Since distributions of types 1 and 2 are the only ones in the above list that have no preferred orientation, comparison with them can be used to establish that an unknown population does have a preferred orientation. Since one of the problems in the present study is the detection of a preferred orientation for the pebble axis populations sampled, comparison with type-1 and type-2 distributions is a useful technique. Simulated samples from such distributions were produced for comparison with the real fabric samples, and are illustrated

in Figure 5.

In comparison with uniformity over the hemisphere, observed values on a point density diagram may be assigned a definite probability using the binomial distribution or the Poisson approximation to it. It is felt that such a technique is of limited usefulness, however, and that a qualitative visual comparison of the point density diagrams is just as effective and more versatile. The procedure described by Flinn (1958) was designed for testing samples against the model of uniformity over the hemisphere, but it requires the use of a particular method of contouring not used here.

Random samples from a uniform distribution over the hemisphere were obtained in the following way. A 10-inch rectangular grid was superimposed on a 10-inch diameter equal-area projection. Points were plotted on the projection using two-digit rectangular coordinates obtained using a table of random numbers. Points falling outside the circumference of the projection were ignored, and the procedure was continued until 50 points lay within the circle of the projection. The trends and plunges represented by the points were then read off using an equal-area or Schmidt net, and a point density diagram prepared in the same way as for the fabric samples.

The population sampled consists of the points defined by all possible pairs of two-digit coordinates, that is, all possible sequences of four digits. Since these are the non-negative integers from 0 to 9999 inclusive, each has the same probability of being obtained from the random number table. Hence the population consists of 10,000 points distributed uniformly over the grid, and each has the same probability of being selected for the sample. Since the projection is equi-areal

the points within the projection represent points distributed uniformly over the reference hemisphere. In this way, samples of 50 random orientations were obtained. They are illustrated in diagrams 505, 506 and 507 (fig. 5).

A simple model of a distribution of type 2 consists of a uniform distribution over that part of the sphere less than a specified angular distance from a given plane. In a real girdle distribution, the probability density would be a maximum in the plane, decreasing away from the plane. Since this is not easily simulated, the uniform distribution was chosen, using a limit of 30 degrees on either side of the plane. This limit was drawn on the projection (using the plane of the projection as the plane of the girdle) and the above-described procedure for obtaining randomly distributed points was followed until 50 points lay within the annular area representing the girdle. Five such samples were obtained; they are illustrated in diagrams 508-512 (fig. 5).

A X^2 test was used to check the randomness of the above samples, from both distributions. For the random distributions over the entire hemisphere both the azimuthal distribution and the polar distribution (frequency in concentric equal areas) were tested. For the girdle distributions, only the azimuthal distribution was tested. Two of the X^2 values were significant at the five per cent level: the polar distribution in sample 506 has a X^2 value with a probability of less than one per cent, and the azimuthal distribution of sample 512 has a X^2 value with a probability of slightly greater than one per cent.

Samples of 100 points from the girdle were obtained by combining sample 508 with sample 509, and sample 510 with sample 511. These are labelled 513 and 514 respectively (fig. 5). Clearly, field samples of

100 axes must be compared with test samples of 100 orientations for the comparison to be valid. All 250 points from the uniform girdle were then run as one sample; this diagram is labelled 515 in Figure 5.

Comparison of Successive Samples

An alternative method of evaluating the random and non-random elements is the selection of several independent samples from the unknown distribution and comparison of these with one another. The non-ramdom element, that is, the density distribution of the population, is common to all the samples, whereas the superimposed random "noise" will be different for each sample. Comparison of several independent samples enables the common features to be singled out. This may be called a test of "reproducibility" of the samples. It is of particular value when the population under study departs only slightly from uniformity, so that comparison with artificial samples from uniform populations reveals no great deviation. In such a case, the reproducibility of the slight non-randomness may be regarded as significant.

This technique is of particular value also when no assumptions can be made about the probable distribution in the population, or when the models that would provide the most useful comparisons cannot be easily simulated. For example, comparison with random samples from a model of a uniform girdle (that is, one having a uniform azimuth distribution in the plane of the girdle) may indicate that the population under study has a non-uniform girdle. Further tests of this sort would require use of models of different kinds of non-uniform girdle; such distributions, however, are not easily simulated. In this case the form of the distri-

bution must be established by comparison of successive samples.

Some valuable information about the adequacy of the 50-axis samples used in this study is provided by the duplicate samples taken at sites 34, 35 and 37 at location F (fig. 11). Each of these samples was initially plotted as two groups of 50, using the first 50 measurements and the second 50 measurements, to test the reproducibility of a 50-axis sample. These plots are shown in Figure 11 as subgroups 1 and 2. As pointed out by Flinn (1958, p.526) this test of the reproducibility of a sample requires the knowledge or the assumption that the fabric, over the distance from which the sample was taken, is homogeneous; only then can differences between successive groups of observations be interpreted as due to sampling rather than spatial variation of the fabric. This difficulty can, however, be overcome by a procedure employed by Kauranne (1960). This involves placing all the even-numbered observations in one group and all the odd-numbered observations in a second group, then comparing these two groups. Any spatial change of the fabric will thus equally affect both groups, and the two groups are truly duplicate samples of the <u>same</u> fabric. Hence any differences can be interpreted as being due solely to sampling. The groups obtained in this way are shown in Figure 11 as subgroups 3 and 4.

The differences between subgroups 1 and 2 are not noticeably greater than the differences between subgroups 3 and 4 at either of sites 34 or 35, suggesting that the fabric at these sites was actually homogeneous throughout the volume of till from which each sample was taken. The difference between subgroups 1 and 2 at site 37 does appear to be somewhat greater than the difference between subgroups 3 and 4, suggesting that the fabric was not in fact homogeneous throughout the

volume of the diamicton from which the measured pebbles were taken. This is not altogether surprising, since it supports the stratigraphic assignment of site 37 to the basal diamictons of the Lake Edmonton sediments instead of to the upper till.

The reproducibility of the samples will be evaluated by comparing subgroups 3 and 4 for each site. At site 34 the maximum that appears at 024 10 in subgroup 3 and at 023 10 in subgroup 4 is clearly reproducible with fairly high accuracy. The two smaller maxima lying on either side of the principal maximum and having somewhat larger plunges are also reproducible but with lower accuracy.

Samples 35-3 and 35-4 both indicate a preference for north-northeasterly and south-southwesterly trends. The grouping here is evidently not as tight as that at site 34, there being no prominent reproducible maximum in the samples of 50 axes. Only the general preference for north-northeasterly and south-southwesterly trends is reproducible.

At site 37, the reproducible features are the three groupings trending northwest, east-northeast and south-southwest. The positions of the density peaks are not constant.

The above experiment shows that a sample of 50 axes is usually sufficient only to indicate approximately the preferred orientation or orientations. Only very high densities in the sample can be regarded as accurate indications of a preferred orientation in the population. It is interesting to note that while the peak at 023 10 in sample 34-4 and the peak at 207 12 in sample 35-3 are comparable in both extent and magnitude, the former is accurately reproduced in the duplicate sample whereas the latter is not. In both cases the maximum density exceeds 15 per cent in a three per cent area. Densities of this

magnitude appear in three of the five samples taken randomly from a uniform girdle.

Thus a density of this magnitude may be closely reproducible or not reproducible at all; only additional observations can determine this. Hence, assuming the criteria of pebble selection used in this study, 50 measurements are barely adequate to define the pebble fabric of a till.

Summary

In view of the preceding discussions, interpretation of the fabric patterns represented by the 50-axis samples discussed below will be done using the following procedure.

First the three per cent density diagram of the sample is compared with the test samples from a uniform distribution and a uniform girdle. If it appears that the fabric is not uniform and does not consist simply of a uniform girdle, then the observations are assumed to be grouped about one or more axes. The problem then is to determine the number of preferred axes; comparison with the duplicate samples from sites 34, 35 and 37 is found to be useful in this step.

Fabric at Exposure

Representation of the fabric of a till unit at an exposure by a single one-point sample is valid only if the fabric is homogeneous throughout the unit at the exposure. Homogeneity of the fabric of a till unit at any exposure implies that the probability of finding a pebble with any given orientation is the same at every point in that

unit at the exposure. An attempt was made to test the homogeneity of the fabric of the upper till at location F (fig. 12). The till is exposed on a slope with a northward dip of about 18 degrees that was made in preparation for mining of the underlying Saskatchewan Gravels. There was no visible evidence of any mass downslope movement.

Fabric samples were taken from the upper till at the nine sites indicated in Figure 12. The samples were grouped both horizontally and vertically in order to detect any systematic variation in the fabric either vertically or laterally. Clearly the patterns shown by the samples differ from place to place. The problem is to determine to what extent these differences are due to "random noise" and to what extent they are due to differences in the fabric.

The fabric patterns are interpreted using the principles established in the above discussion. Comparison of sample 40 with the test samples (fig. 5) shows two things: (1) the observed axes are concentrated close to a near-horizontal plane, and (2) in this plane, the direction 045 14 is definitely preferred; the grouping about this direction is so tight as to suggest that the two smaller groups at 100 11 and 150 00 are not merely part of the "tail" of this grouping but in fact indicate one or more other elements in the fabric pattern. Whether they are simply accidental groupings in the preferred near-horizontal plane, or whether they indicate two more preferred orientations cannot be established. The maximum dip of the preferred plane, as determined from the vector magnitude diagram, is 14 degrees toward 037 degrees (Table 2).

The axes in sample 38 are concentrated close to a plane dipping 29 degrees towards 039 degrees (Table 2). Comparison with test samples

508-512 indicates no great deviation from the patterns expected from a uniform girdle. Sample 42 is similar to sample 38 in being concentrated close to a plane but having no reliable indication of a preferred direction. The preferred plane dips 29 degrees towards 011 degrees.

The axes of sample 6 are concentrated close to a near-horizontal plane (the position of the minimum on the vector magnitude diagram indicates that this plane dips 5 degrees towards 125 degrees). In this plane there is clearly a preference for the position indicated by the density maximum at 066 00.

Comparison of sample 34 with test samples 513 and 514 (fig. 5) indicates a preference for a near-horizontal plane (dipping 07 degrees towards 055 degrees) and a clear preference for northeasterly and southwesterly trends. As discussed previously, the constancy of the position of the density maximum at 024 10 in independent samples of 50 axes from this site strongly suggests that this is a preferred orientation of the till pebbles at this site. The sample suggests that there is a second preferred orientation near the density maximum at 099 05, but its position is indicated only approximately.

The axes of sample 35 are clearly, though loosely, grouped about a near-horizontal line trending about 026 degrees. The vector magnitude diagram indicates that the preferred plane dips 53 degrees towards 295 degrees, but this plane is only weakly preferred.

Sample 36 shows a preferred plane that dips 29 degrees towards 038 degrees. The double-peaked grouping in the northern part of the diagram probably represents a preferred direction.

Sample 39 shows a preferred plane dipping 14 degrees towards 079 degrees. The gaps in the girdle at 080 and 118 degrees and the low at

006 degrees suggest a sparsity of pebble axes in the population with north-south or east-west trends, that is, a preference for northeast-southwest and northwest-southeast trends.

Sample 41 also has a preferred plane, dipping 05 degrees due north. In this case the two gaps in the girdle at 060 and 155 degrees suggest preferences for the east-southeasterly and north-northeasterly parts of the girdle.

Thus the nine samples indicate preferred orientations ranging from north to east-northeast. If the degree of reproducibility demonstrated for sites 34 and 35 (fig. 11) is assumed for all nine sites, then a substantial variability of the fabric must be inferred. The implication of this conclusion is that a one-point sample from the upper till at Location F is likely to give misleading information if it is used to infer ice movement direction. Sampling procedures designed to overcome this difficulty are discussed below. It is interesting to note also that there appears to be no greater consistency of the fabric horizontally than vertically.

The approach of using a single one-point sample to represent the fabric of a till at an exposure may be further evaluated by considering other locations at which several samples were taken from the same unit. Reasons for the differences or similarities of the samples are not discussed; it is only the fact of the variability that is of interest here. Discussion of fabric origins will be found in a later chapter.

Eleven samples of 50 axes are available from the till exposed at Location B (fig. 10). Considerable variability is evident in both units. Even the three samples from the top of the till include both northwest-southeast and northeast-southwest preferred orientations. Thus the same

high variability is present laterally as vertically.

Samples 4 and 5 (fig. 6) were both taken from the lower till at Location E, about 200 feet apart. The lower till is about 12 feet thick; sample 4 is from the top two feet, and sample 5 from three to four feet above the base. The two samples bear no resemblance whatever.

Samples 7 and 21 (fig. 6) from the base of the lower till at Location F were taken about 300 feet apart and are quite similar, both showing a preference for a northeast-southwest trend.

At Location G (fig. 7) the writer originally obtained samples 31 and 32, respectively from the lowermost two feet and the uppermost two feet of the 14 feet of upper till, sample 32 being vertically above sample 31. No similarity is evident. Just over a year later, sample 466-5 was obtained by another worker from the same gravel pit at about 10 feet below surface. The exact site is almost certainly at least 100 feet from sites 31 and 32. Assuming seven feet of Lake Edmonton sediments as mapped by the writer, site 466-5 would be about three feet below the top of the upper till, and thus stratigraphically close to site 32. The similarity of samples 32 and 466-5 is striking.

Samples 8 and 9 (fig. 7) from Location H were taken from two different "phases" of the lower till that are easily distinguishable in the field (Plate 2-G). Phase A (sample 9) is dark grey and has a fractured, blocky structure. Phase B (sample 8) is brown and has a characteristic granular structure. Samples 8 and 9 are clearly very different. However, no information is available concerning the variability of the fabric within each "phase" of the lower till at this exposure.

At Location I (fig. 9) the lower "till" consists of three members

separated by bedded sands, silts and clays. The entire lower till complex is 23 feet thick. Sample 18 is from the middle member (4 feet thick); sample 26 is from the lower member (6 feet thick) directly below sample 18; sample 44 is from the middle member about 10 feet horizontally from sample 18. The similarity of all three of the above samples is remarkable.

Samples 22 and 23 (fig. 9) from the lower till at Location K at first sight appear dissimilar, but actually are alike in having a fairly high proportion of observations trending east-northeast or west-southwest.

Although differing in detail, samples 466-3, 466-8, and 466-11 (fig. 13) from the upper till at Location L all indicate a general preference for a northeast-southwest trend. Sample 466-1 differs in having prominent maxima in both the northeast and the north-northwest. For the lower till at the same location, samples 45 and 466-10 (fig. 13) both indicate a preferred west-northwest trend. Sample 43, however, differs greatly from 45 and 466-10.

Thus it can be seen that, although different samples from a till unit at a given exposure are in some cases very similar, this can by no means be counted upon, since frequently great variation is evident. It must be concluded that a single one-point sample should never be depended upon to indicate the preferred trend of elongate pebbles in a till unit at an exposure . Indeed, the assumption that the pebbles in a till unit at an exposure have a single preferred trend must be questioned. They may have many preferred trends, or they may have different preferred trends in different parts of the exposure, both laterally and vertically.

Effect of Pebble Size and Shape

Possible effects of pebble size and shape on the orientation of the a-axis were looked for by measuring the lengths of the a-, b- and c-axes of 226 pebbles from sites 34, 35 and 36 in the upper till at Location F. These dimensions, together with the a:b and b:c ratios and the attitude of the a-axis are given in Appendix C. In addition, the dimensions and attitudes of a number of flat pebbles with low elongation were measured. Appendix C also gives the shape classification of the pebble according to the system of Holmes (1941).

Two variables were examined for a relationship with the a-axis orientation: the a:b ratio (here called "elongation") and the a-axis length. Pebbles were grouped according to their a:b ratio and point density diagrams prepared for the a-axes of each group (fig. 14). The top row of diagrams in Figure 14 shows the effect of raising the lower limit of the a:b ratio. As the lower limit is raised from 1.15 to 1.25 the maximum at about 024 14 becomes stronger, but no other changes in the a-axis distribution are evident. As the lower limit of a:b ratio is raised from 1.25 to 1.35 the maximum at about 024 14 becomes weaker, and the grouping centred at about 351 25 becomes stronger. These two trends continue as the lower limit of a:b ratio is increased to 1.45 and then to 1.55 at which point the maximum at 349 19 is stronger than that at 029 15. In addition, southwest-trending axes appear as a separate group centred at about 208 25, and the grouping that was evident at about 055 27 has disappeared completely. Thus there appears to be a tendency for the north-northwesterly and south-southwesterly trending

maxima to be more strongly preferred by pebbles with higher a:b ratios. It is worth noting that the weak maximum at about 104 09 on diagram 106 (a/b ≥ 1.15) survives right through to diagram 124 (a/b ≥ 1.55) and still trends 104 degrees.

The bottom row of diagrams on Figure 14 illustrates groups with a:b ratios restricted to the ranges 1.15 to 1.25, 1.25 to 1.35, 1.35 to 1.45, and 1.45 to 1.55. If diagram 121 is compared with test samples 505-507, it appears to be random. On the other hand diagram 122 shows a preferred orientation of 017 17. This suggests that 1.3 would be a better lower limit of a:b ratio than 1.2 for use in future studies. However, the question of a:b ratio clearly requires further study, since it appears that the a:b ratio can affect the orientation of the pebble. A possible explanation for the relationship between a:b ratio and a-axis orientation observed here is discussed below in the section dealing with the movement of the glacier that deposited the upper till. Johansson (1968) also classified according to a:b ratio pebbles from till deposited by a glacier whose movement direction had changed. However, his data indicate no apparent correlation between a:b ratio and a-axis orientation.

Figure 15A illustrates the effect of a-axis length on a-axis orientation. Three lower limits of a-axis length, 5mm, 25mm, and 40mm, were used, and these subgroups studied using two different lower limits of a:b ratio, 1.15 and 1.45. The top row of diagrams in Figure 15A illustrates the effect of an increasing lower limit of a-axis length using all pebbles with a:b ≥ 1.15. As the lower limit of a increases, the maxima become stronger (except the small one at 104 09), but no change in the relative magnitudes of the maxima is evident. The same effect is seen in the bottom row where a:b ≥ 1.45; here the strengthening of the

maxima is even more pronounced. Again, no substantial change in relative strengths of the maxima is seen. Thus, while there is no apparent relationship between the size of a pebble and the orientation preferred by that pebble, larger pebbles do display a far smaller amount of scatter than do smaller ones. Clearly, the sizes of pebbles measured for a fabric sample will therefore partially determine the minimum number of observations required, since the more scatter there is in the observed orientations the more observations are necessary. Johansson (1968) classified till pebbles according to size. His data indicate that larger pebbles have more scatter in their orientations than smaller pebbles. This difference may be due to the fact that the pebbles measured by Johansson were large compared to those used in the present study and in higher concentration, so that considerable interaction of pebbles probably took place during glacier movement.

Figure 15B shows the distribution of the c-axes (or poles of the a:b planes) of 84 "flat" pebbles from the same three sites -- 34, 35, and 36 -- in the upper till at Location F. The highest density is very close to the centre of the projection, indicating that the most strongly preferred attitude of pebbles with high b:c ratios (that is, "flat" pebbles) is with the a:b plane nearly horizontal. A similar observation was made by Harrison (1957) and Johansson (1968). Diagram 128 in Figure 15B shows a second grouping of c-axes with plunges from 18 to 34 degrees and trends from about 090 to about 125 degrees. These pebbles have their a:b planes dipping steeply to the northwest and striking from northeast to north. A possible explanation for this grouping of c-axes will be discussed below in relation to the movement directions of the glacier that deposited the upper till.

Recommended Sampling Procedures

The above data illustrating the variability of fabric pattern over short distances within a single stratigraphic unit clearly indicate that the approach of using a one-point sample to characterize the fabric pattern of a unit at an exposure is not valid. The data support the similar conclusions of Kauranne (1960), Andrews and Smith (1966), and Andrews and King (1968).

The question thus arises of what valid approach can be taken to the problem of measuring till fabrics. Clearly, if single-point samples are inadequate, an approach involving multiple-point sampling must be adopted; that is, many small samples from different parts of an exposure must replace a single large sample from one point in the exposure. Andrews and King (1968) suggested such multiple-point sampling and the use of Watson and Irving's (1957) procedure involving calculation of within- and between-site precision parameters and to determine the optimum sampling pattern. As was pointed out earlier, this method is not applicable to till fabric data since the latter cannot be described adequately by Fisher-type models. The problem of how many points should be sampled and how many pebbles measured at each site must be solved by some other means.

To this end, some experiments in sampling were performed using the data available from the upper till at Location F. A total of 550 long axes were measured at nine sites (fig. 12). Although substantial variability of the fabric pattern was evident from site to site, the density diagram for the whole group (number 118, fig. 12) showed a single maximum

at 029 10, coinciding with the trend of groove molds (Plate 1-A, 2-E) from 028 to 033 degrees seen on the base of the upper till at several localities within one mile of Location F. This coincidence suggests that the whole group does indicate the dominant ice-movement direction in spite of the inhomogeneity of the fabric apparent from the individual samples. Although it cannot be assumed that this relationship holds at other localities and for other till units, it was used as a basis for experimenting with some possible sampling patterns.

First, samples of 50 axes were made up by taking 10 from each of the five sites 39, 40, 35, 41, and 42 (fig. 12). The first 10 observations from each site were used to make the first sample, the second 10 observations from each site to make the second sample, and so on. The resulting samples are numbered 101-105 and appear in Figure 16. Sample 101 shows no clear indication of non-randomness in its near-horizontal girdle when compared with test samples 508-512 (fig. 5). Samples 102-105 show a more or less definite preference for northeasterly and southwesterly trends, but the trend of the strongest maximum ranges from 028 to 048 degrees. Thus these samples do not provide a reliable estimate of the maximum of the whole group at 029 10.

For the next experiment, the sample size was increased from 50 to 80 and all the available sites included. To make each experimental sample, 10 pebbles were taken from each site except 6 and 34, each of which was represented by only five observations since they lie close together. Again, five samples were constructed; they are shown as samples 131-135 on Figure 16. Only three of the five, 132, 133, and 135, indicate a preference for a northeasterly trending orientation. Number 131 shows a girdle dipping slightly to the northeast, but could not be

interpreted as indicating any preferred orientation. Number 134, if seen alone, would be interpreted as indicating two preferred orientations, the stronger dipping slightly toward 350 degrees, the other dipping slightly toward 080 degrees. It would be reasonable to infer from this sample ice movement from 350 degrees, regarding the maximum at 080 degrees as representing transversely oriented pebbles. Thus a single sample of 80 long axes taken from the nine sites used here cannot be depended upon to indicate a preference for a northeasterly trending orientation.

Finally, the sample size was increased to 200, 25 observations being taken from each site except 6 and 34. 25 observations were taken from sites 6 and 34 together, the split being first 12 to 13, then 13 to 12. The resulting samples are numbered 136 and 137 on Figure 16. Comparison with test sample 515 (consisting of 250 axes; fig. 5) shows that both samples 136 and 137 indicate (1) a girdle dipping slightly to the northeast, (2) a preference for a northeasterly trending orientation, and (3) a high degree of scatter, the scatter being greatest in the plane of the girdle. The strongest maximum of sample 136 is at 042 22; that of sample 137 is at 040 12. These two samples are thus very similar; they are also similar to the total group (sample 118, fig. 12) except for the difference (11 to 13 degrees) in the trend of the maximum.

It is concluded that a sample of less than about 200 long axes from the same nine sites could not be relied upon to indicate the northeasterly trending preferred orientation. Fabric and other evidence lead to the conclusion that the fabric of the upper till at Location F is complex, that is, it has been produced by ice movement in more than one direction (in this case ranging from northeast-southwest to north-north-

west-south-southeast). This may be the cause of the high variability of the fabric over short distances that in turn results in the high scatter observed in the total group. This high degree of scatter is the reason why large samples are necessary to define the preferred orientation when the sample is spread over the whole exposure. Thus the complexity of the fabric at this exposure might be cited as the underlying reason for the necessity of such large samples, and the exposure regarded as a special case. However, it can never be assumed that an unknown fabric is not such a complex fabric. Few till fabrics have been studied in sufficient detail to establish either their complexity or their variability. Therefore, the worst must be expected at each new exposure, and the complexity and variability of the fabric tested. A decision concerning the minimum adequate size of the total sample should then be made on the basis of this information.

Rigorous methods for determining reliability of sample means and adequate sample sizes have been devised (Dennison, 1961) but are applicable only to unimodal distributions and are probably of little value for till fabrics. The method of Harris (1969) for determining "minimum significant orientation count" may be a useful technique. It involves the use of <u>sequential sampling</u> (Pincus, 1953, p.487) to estimate minimum satisfactory sample size. Pincus (1953) recommended use of a system of sequential sampling in situations involving more than one concentration of orientations.

One possible procedure involving sequential sampling at a new till exposure might be as follows. A sample of 100 axes is taken, measuring 20 at each of five sites located, perhaps, at the horizontal and vertical extremes of the exposure with one in the centre. The trends of the first

50 axes (that is, the first 10 from each site) are plotted on prepared polar coordinate paper. Similar plots are made of the first 75 axes (the first 15 from each site) and of the whole group of 100 axes. By comparing these plots, the investigator can decide whether or not the features of the fabric pattern in which he is interested have been established with the desired degree of reliability. If not, additional sets of five axes from each site can be added to the whole sample until the investigator is satisfied that the relevant aspects of the fabric have been determined with sufficient accuracy.

A procedure such as this ensures that sufficient measurements will be made in the field to determine the required information. In addition, valuable data are obtained concerning the degree of variability of the fabric. If sample sites are judiciously chosen, independent information can be obtained about horizontal and vertical variation. The number of sites should ideally be related to the variability of the fabric; if an operator began with five sites and found great variability, he might be wise to add four more sites to form a grid of nine.

It is emphasized that the significance of such a multiple-point sample when the fabric is variable is by no means established. The coincidence of the trend of the density maximum of the total group and the trend of basal groove molds for the upper till at Location F cannot necessarily be extrapolated to other locations or other units.

The data examined here indicate that pebbles with a:b ratios of less than about 1.3 are not good indicators of ice-movement direction; more elongate pebbles should therefore be used. In addition, the a:b ratio of each pebble should be recorded, since pebbles with different

a:b ratios may have different preferred orientations.

The sizes of pebbles used for a fabric study should also be recorded, so that any correlation between size and preferred orientation or degree of preferred orientation can be discovered. This information could save much unnecessary work and improve results in later stages of the investigation.

Work Needed

Extensive investigation of small-scale pebble fabric variability in tills is clearly necessary. Suitable sampling procedures for determining the overall fabric of a till exposure must be established, and the meaning of such an overall fabric when the fabric is variable must be investigated.

The a:b ratio of a pebble apparently has some effect on its orientation in a till. It was suggested above that more elongate pebbles achieve their "stable" orientation more rapidly than less elongate pebbles, so that when the direction of movement of a glacier changes the more elongate pebbles are the first to be re-aligned. Further investigation of this possibility is required, since it could prove to be a powerful tool in the determination of the movement history of glaciers.

SUMMARY OF RECOMMENDATIONS

1 Fabric variability

Till fabrics are locally variable, and a spatially limited sample (e.g. 2'x2') may be grossly misleading if taken as indicating the dominant movement direction of the glacier at that locality. Pebbles should be taken from several points spaced as widely as possible across the exposure to estimate fabric variability. Combining them into a single sample may indicate the dominant movement direction.

2 Size of pebbles

Present study indicates larger pebbles have less scatter in orientation than smaller pebbles. Hence the largest available stones should be used for determining ice movement direction. The sizes of all stones used should be recorded.

3 Shape of pebbles

Present study indicates a:b ratio may affect orientation of pebble. A pilot study should be carried out in which pebbles are grouped according to size and a:b ratio and possibly other axial ratios, so that any relationship between shape and orientation can be detected. The most suitable combination of axial ratios and size for the purpose of the study can then be selected, and the efficiency of subsequent work greatly increased. The present study indicates that pebbles with a:b ratios less than about 1.3 are not good indicators of ice movement direction, but this may not be true in other areas. The axial lengths of all

pebbles used should be recorded.

4 Facetting

Many glacial stones have a "pointed end" and a "blunt end". The direction in which the pointed end of such a stone points should be recorded, as this will help to determine the sense of ice movement.

5 Pebble density

The density of pebbles in the till may affect the reliability of the pebble orientations as indicators of ice movement direction, and an estimate of the pebble density should be recorded at each sample site.

6 Sample size

Minimum adequate sample size will vary according to fabric variability, amount of scatter in pebble orientations, and other factors. It should be determined by a procedure of sequential sampling at each sample site.

ICE MOVEMENT DIRECTIONS AND FABRIC PATTERNS OF GLACIAL SEDIMENTS IN THE EDMONTON AREA

Local Stratigraphy

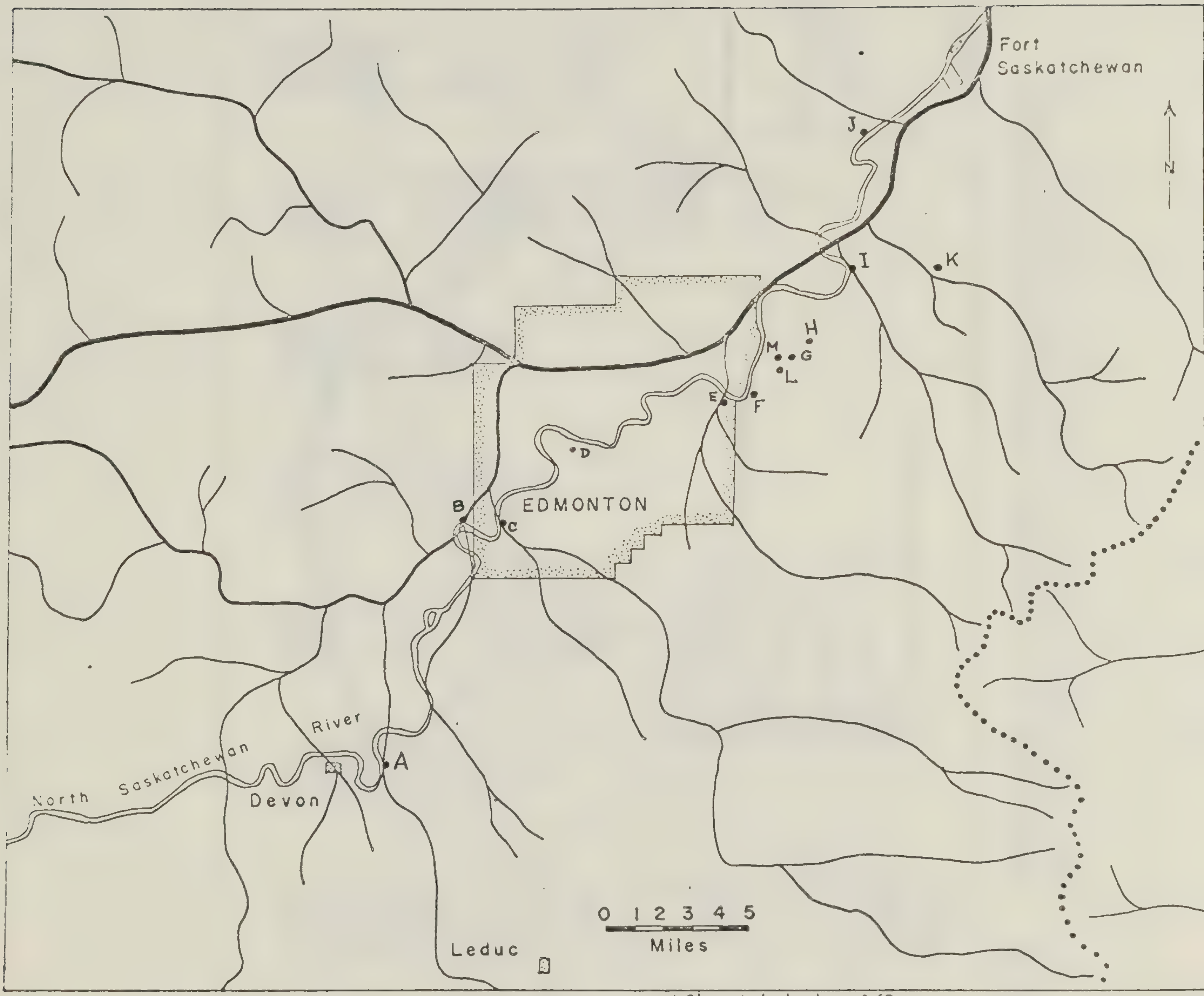
Bedrock

The bedrock of the area consists of poorly consolidated Upper Cretaceous bentonitic shales and sandstones with some coal seams and bentonite beds. The beds dip at about 20 feet per mile to the southwest (Westgate, 1969). Thalwegs of the major valleys in the bedrock surface are shown in Figure 17.

Quaternary Deposits

The succession of Quaternary deposits in the area is shown in Figure 18. These deposits have been described in detail by Westgate (1969). The Quaternary landforms and surficial deposits of the district, shown in Figure 19, have been described by Bayrock and Hughes (1962).

The oldest Quaternary deposits are fluvial gravels and sands known as the Saskatchewan Gravels. Vertebrate fossils suggest that the youngest beds of this formation are of late Pleistocene age (Reimchen, 1968). A greyish-brown, dense, clay-loam till with some inclusions of stratified sand overlies the Saskatchewan Gravels or lies directly on bedrock. It varies in thickness from zero to more than 20 feet, is highly jointed, and possesses some folded joint surfaces. The exact age of this



After Westgate 1969, mod. After V. Carlson 196

Figure 17 Thalwegs of preglacial valleys in the Edmonton area and locations of measured sections.

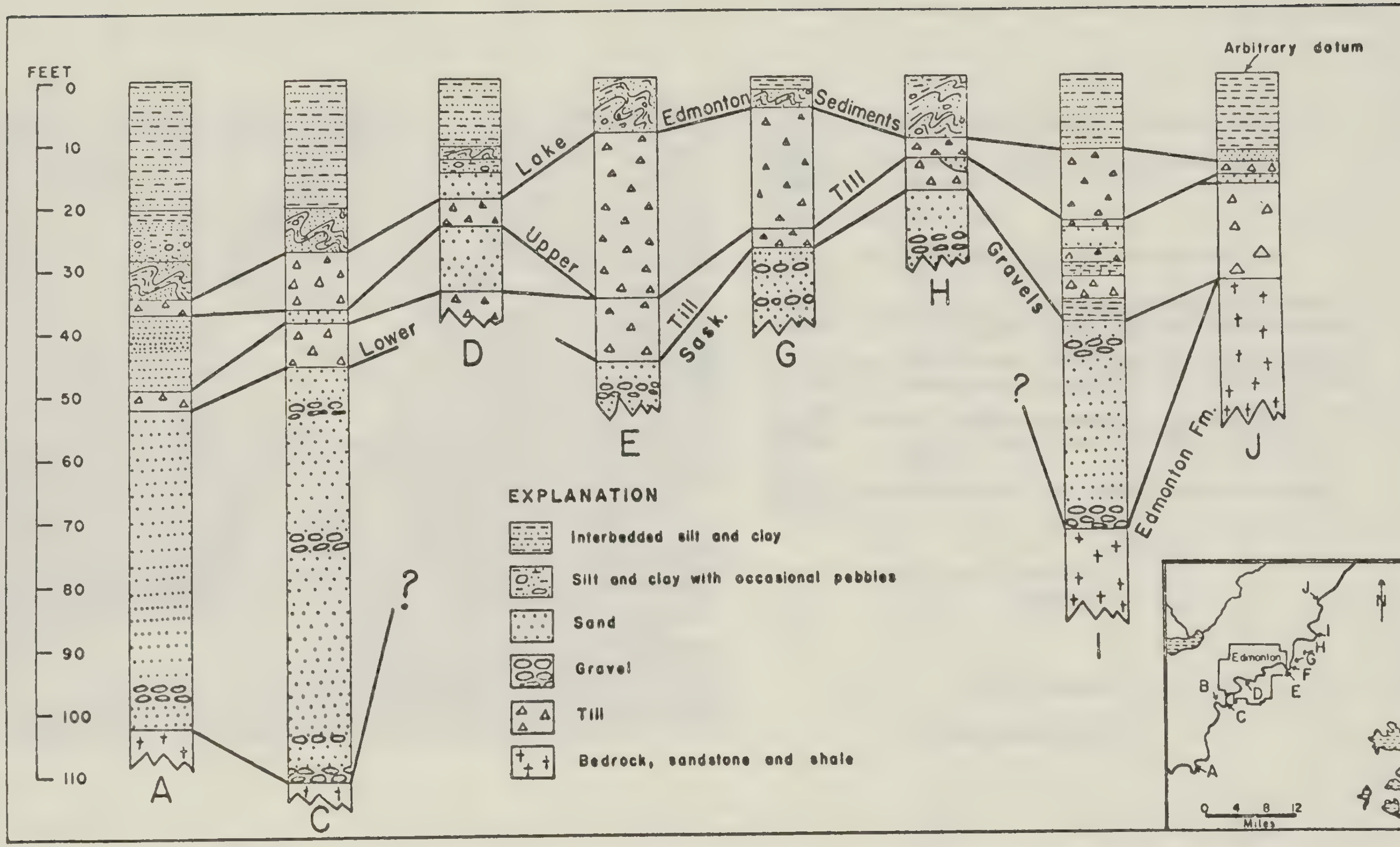


Figure 18 Stratigraphy of Pleistocene deposits exposed along the North Saskatchewan River valley (after Westgate, 1969).

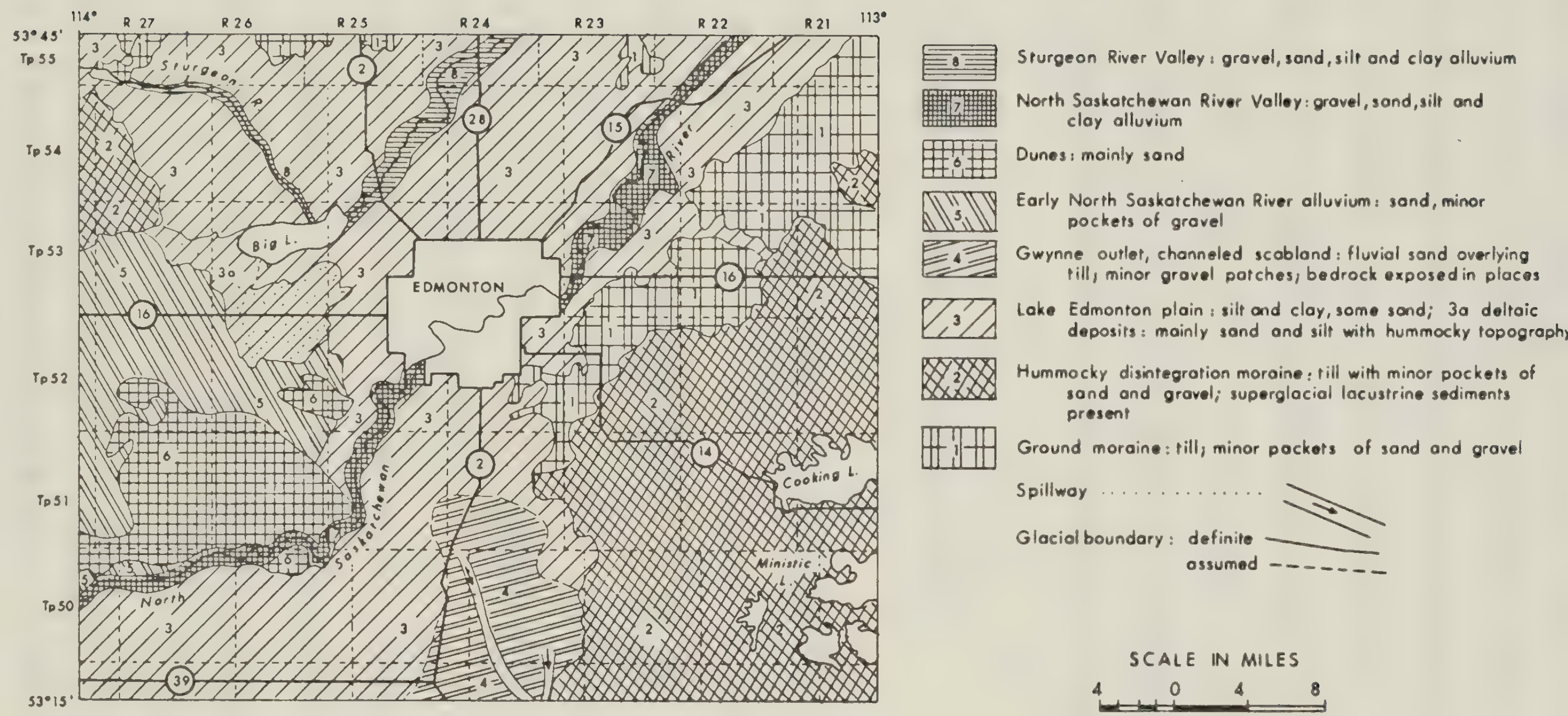


Figure 19 Quaternary landforms and deposits of the Edmonton area (after Westgate, 1969, modified after Bayrock and Hughes, 1962).

lower till is not known but it is probably of early Wisconsin age.

Stratified sediments, 40 feet thick in places, and referred to as the Tofield Sand, commonly separate this till from an overlying till. This upper till, which is loamy and yellowish brown, has a pronounced columnar structure; the greatest observed thickness is 25 feet. This till is believed to be of late Wisconsin age.

Up to 50 feet of well-bedded Lake Edmonton sands, silts and clays rest on the compact upper till. In some areas, however, deformed stratified beds with lenses of till-like material, interpreted as ablation deposits, immediately underlie the lacustrine sequence (fig. 18).

Stratigraphic Setting of Fabric Samples

Detailed location and stratigraphic information for all sections will be found in Appendix B, and the fabric diagrams appear along with simplified stratigraphic sections in Figures 6, 7, 9, 10, 12, and 13 (in pocket). In this section, all other relevant information, such as structural observations, will be listed, and consideration given to stratigraphic interpretation where necessary.

Fabric samples were taken mostly by the writer from 41 sites at 11 locations (A to L except C, fig. 20). These samples (and corresponding sites) are numbered 1 to 45 with the numbers 14, 15, 16 and 19 not used. 12 more samples, including one from an additional location, M (fig. 20), were obtained as part of a research project by students in the Geology 466 class at the University of Alberta in the fall of 1969, and are numbered 466-1 to 466-12.

Each sample will be assessed to establish what reliable inferences

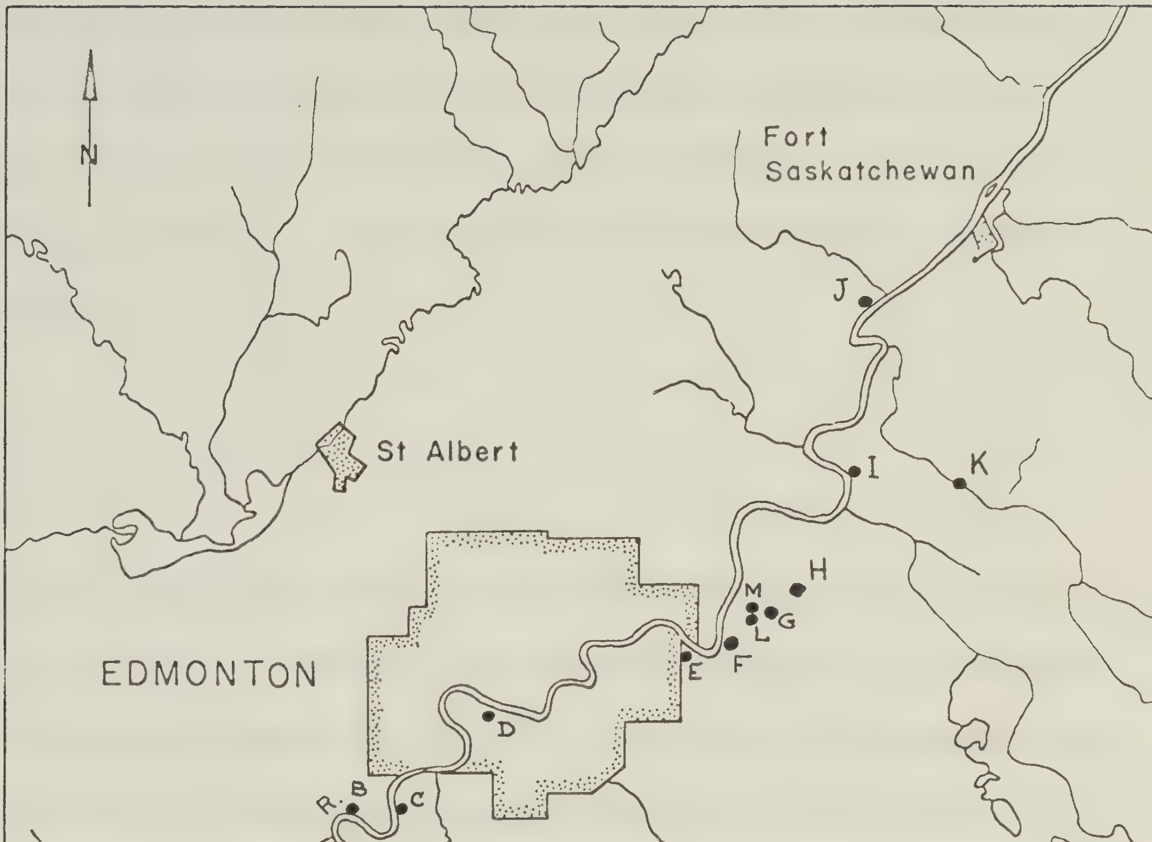


Figure 20 Locations of measured sections.

can be made about the distribution of pebble axis orientations at that site. The degree of reliability of such inferences will also be considered. The techniques used are: (1) Comparison with test samples from a uniform distribution and a uniform girdle. (2) If the distribution is not similar to either of the two models considered in 1, comparison with samples 34, 35 and 37, each of 100 observations, or with subgroups of these, is found helpful in establishing the form of the distribution.

Location A

Both upper and lower tills are present at this section, being separated by 12 feet of Tofield sand, but only the fabric of the upper till was determined (sample 12, fig. 6). If sample 12 is compared with the test samples from hypothetical models shown in Figure 5, it is readily seen that it differs from samples 505, 506 and 507 from populations distributed uniformly over the hemisphere in lacking orientations with steep dips. In this respect it resembles samples 508-512 from a uniform girdle. However, sample 12 has a density of more than 8e in a three per cent area (where e is three per cent, the expected density for a uniform distribution), while the highest density in samples 508-512 (all using a three per cent area) is less than 7e and this appears only once. Apart from this high density, sample 12 resembles samples 508-512. It is inferred that the a-axes of pebbles at site 12 are concentrated into a near-horizontal plane, and in this plane have a preference for north-northeasterly and south-southwesterly trends. It is difficult to make any inferences about the strength of the preferred orientation.

If a density of more than 6e in a three per cent area is to be expected in a sample of 50 orientations from a uniform girdle, then probably only a slight preference for one orientation in that girdle could result in a density of 8e to 9e. In other words, the degree of preference may be weak or strong; 50 measurements are insufficient to determine this.

Location B

At this location approximately 50 to 55 feet of till is found between the underlying Saskatchewan Gravels and the overlying Lake Edmonton sediments. Although both upper till and lower till appear to be present here, the unconformity is not clearly visible and its placement at 10 feet below the base of the Lake Edmonton beds is tentative. The fabric diagrams and simplified stratigraphic section are shown in Figure 10. A total of 11 fabric samples are available from the till; their stratigraphic positions are shown in Figure 10.

Sample 2 shows a preferred orientation indicated by the density maximum at 033 24. There is probably a second, weaker preferred orientation at 330 18. Sample 33 indicates a preference for trends in the northwesterly quadrant, all parts of the quadrant apparently being equally preferred. Sample 466-4 shows a strong preference for the orientation 036 00. Sample 466-6 has a near-horizontal girdle, but no preferred orientation. Sample 466-12 shows a preferred orientation at 008 14, and a weaker preferred orientation at 276 10. Sample 466-7 shows a clear but quite disperse grouping about approximately 068 30. Sample 30, when compared with test samples 505-507 (fig. 5), appears to be non-random on the hemisphere by virtue of the almost complete absence of axes

in a wide band from the northeasterly quadrant to the southwesterly quadrant. The centre line of this band would be the trace of a plane dipping steeply to the northwest; it is therefore inferred that there is a preferred orientation of pebble long-axes dipping slightly to the southeast. This inference is supported by the appearance of a density value of more than 5e at 155 41; a density of this magnitude did not appear on any of the three test samples 505-507. Sample 29 has a near-horizontal girdle; the double break in this girdle trending northwest-southeast and the near break in the northeasterly quadrant suggest a preference for north-south and east-west trends, but this is not well indicated. Sample 28 shows a preference for the position of approximately 001 04. Sample 27 indicates a preferred orientation at 020 00, and sample 466-2 a relatively strong preference for the position 025 12.

Location D

At this location, only the top few feet of the lower till was exposed (fig. 6). It was overlain by 10 feet of sand and silt, which was in turn overlain by a complex, partially deformed sequence of brown diamictons interbedded with silts and fine- to medium-grained sands. Sample 1 was taken from the lowermost diamicton in this sequence. The apparent lateral equivalents of these diamictons a few hundred feet east have a granular texture with granules up to about one half inch, so that they resemble "mud pebble conglomerates". This fact, and the interbedding of the diamictons with silts and fine sands, suggest that the diamictons are lacustrine turbidites and not tills. For this reason sample 1 is not considered in the discussion of ice-movement directions.

The fabric of sample 1 shows a preferred orientation at 351 07 and possibly a weaker one at 057 00. Sample 13 is from the top of the lower till; it indicates two preferred orientations, the stronger at 134 16, the weaker at 021 00.

Location E

Both tills are present at this exposure, with no intervening beds (fig. 6). A large mass of Cretaceous Edmonton Formation is contained within the upper till at one end of the exposure.

The upper till fabric, sample 3, shows a preference for north-easterly trends, possibly for the two positions 008 19 and 066 19. Interpretation of sample 4 is uncertain; it may indicate a preferred orientation at 090 20, may indicate only a preferred plane dipping 28 degrees towards 029 degrees, or may be random. The lower till fabric, sample 5, shows two preferred orientations at 032 05 and 138 05.

Location F

At this locality the upper till is overlain by Lake Edmonton sediments and rests directly on lower till. One fabric sample (37) was taken from the base of the Lake Edmonton sediments (fig. 6), nine from the upper till (fig. 12) and two from the lower till (fig. 6). The samples from the upper till were described in connection with fabric variability. Sample 37 from the Lake Edmonton deposits shows a preference for a near-horizontal plane, and comparison with test samples 513 and 514 (fig. 5) does not justify inference of any preferred direc-

tion in this plane.

Sample 7 from the lower till shows preference for a near-horizontal plane and a well-defined preferred orientation at 044 04. A weaker preferred orientation at 308 03 is suggested. Sample 21, also from the lower till, indicates only a preference for trends in the northeasterly quadrant.

The base of the lower till at this locality consists of a clayey layer that is highly sheared (Plate 2-A, B, C). The top of this highly sheared layer is marked by a prominent, flat, shiny shear plane. Slickensides on this shear plane form two major sets with trends of 033±6 and 000±2 degrees. A less prominent set of slickensides has a trend of about 294 degrees; curved slickensides were also observed.

The base of the lower till is undulated in places. Two adjacent undulations with a wavelength of about two feet had trends of 335 and 337 degrees. A third undulation about 500 feet from the other two had a trend of 310 degrees.

Location G

Samples 32, 466-5, and 31 are from the upper till at this locality (fig. 7). Up to two feet of lower till separate the upper till and Saskatchewan Gravels, but no samples are available from it. Both samples 32 and 466-5 indicate a preference for southeasterly trends with a high degree of horizontal scatter. Sample 31 reveals a preference for low plunges, but no preferred trend.

At one point where the upper till rests directly on sand of the Saskatchewan Gravels, its base forms well-defined groove molds that

trend from 028 to 033 degrees.

Location H

Here, about three feet of upper till rests directly on about three feet of lower till, and is overlain by about 10 feet of Lake Edmonton deposits (fig. 7 and Plate 2-D). The exposed face is about 140 feet long and trends east-west. Fabric samples 10 and 11 are from the Lake Edmonton sediments, while samples 8 and 9 are from the lower till.

Sample 10 shows preferred orientations at 016 06 and 282 05. Sample 11 has a near-horizontal girdle, with two gaps trending northwest-southeast, a maximum at about 273 05, and a weaker maximum at about 224 05.

Groove molds on the base of the upper till (Plate 2-E) trend 035 degrees. Most of the lower till is dark grey, has a fractured, blocky structure and contains numerous irregular masses of clean, fine- to medium-grained sand (Plate 2-F). Part of the lower till at one end of the exposure, however, is brown and has a fine granular texture that contrasts with the blockiness of the rest of the till (Plate 2-G).

Sample 9 (fig. 7) comes from the blocky lower till at the middle of the exposure. It shows a near-horizontal girdle with a preferred orientation of 036 05. Sample 8 was taken from the granular-textured lower till; it also has a girdle with a preferred orientation, but in this case the girdle dips 60 degrees toward 021 degrees, and the preferred orientation is at 322 48. Figure 21 shows the distribution of the poles of 47 joint planes in the blocky lower till close to the site

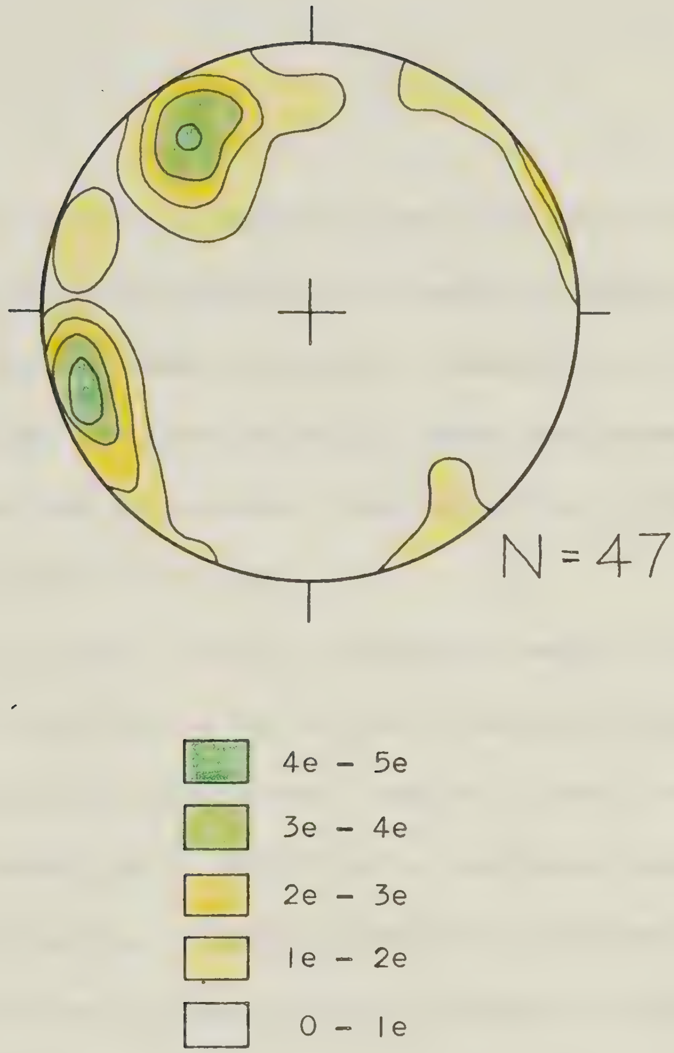


Figure 21. Poles of joints in fractured lower till at Location H.

Diagram prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of poles plotted. North at the top.

of sample 9. It has two maxima at 325 23 and 250 12. There is no apparent relationship between these maxima and the fabric of the lower till.

Location I

At this locality about 11 feet of upper till rests directly on a 15-foot thick sequence of three dark grey diamicton members separated by two sand, silt and clay members (fig. 9). Sample 17 is from the upper till, samples 18 and 44 are from the middle dark grey diamicton, and sample 26 is from the lower dark grey diamicton. The horizontal distance between samples 18 and 44 is 10 feet.

Sample 17 has a girdle dipping 30 degrees toward 205 degrees. There appears to be a preference for west-northwesterly and east-south-easterly trends, though not pronounced. Sample 18 has a relatively strong preferred orientation at 327 29 and a preferred plane dipping 31 degees toward 356 degrees. Sample 44 shows a preference for the orientation 350 11 and for a plane dipping 12 degrees toward 004 degrees. Sample 26 indicates only a preferred orientation of approximately 342 15.

The interbedding of the dark grey diamictons with sand, silt and clay suggests that this part of the sequence is of lacustrine origin. The lower diamicton has a granular structure similar to that seen in the brown diamictons at Location D; this structure is believed to be indicative of a subaqueous gravity mass flow deposit (turbidite). The entire sequence of diamictons, sands, silts and clays between the Saskatchewan Gravels and the upper till is regarded as probably lacustrine in origin.

Location J

Here about two feet of upper till is separated from about 15 feet of lower till by one foot of sand. Sample 20 is from the upper till and sample 25 from the top of the lower till (fig. 9).

Sample 20 has a girdle dipping 10 degrees toward 344 degrees. Northwesterly and southeasterly trends are preferred, with considerable horizontal scatter. Sample 25 has a peak at 346 41, and a smaller peak at 206 11. The "best fitting" plane dips 52 degrees toward 288 degrees. The lack of continuity of the density high suggests that there is not a uniform girdle, but that the density peaks indicate two preferred positions.

Location K

A sketch of this exposure showing its structural features appears in Figure 22. The bedrock surface slopes quite steeply (10-20 degrees) downward to the south. Isolated remnants of massive lower till and of rhythmically bedded brown to buff clay rest directly on the bedrock surface. These are overlain by up to 15 feet of upper till which, in its upper part, contains thin sand beds that are folded as indicated in Figure 22. It is overlain by clean fine- to medium-grained sand. This sand and the folded upper till are truncated by a nearly level unconformity on which rests a diamicton about three feet thick that thins rapidly southward.

Fabric samples 22, 23 and 24 were obtained from the points indicated in Figure 22. Sample 22 (fig. 9) shows a preference for a plane

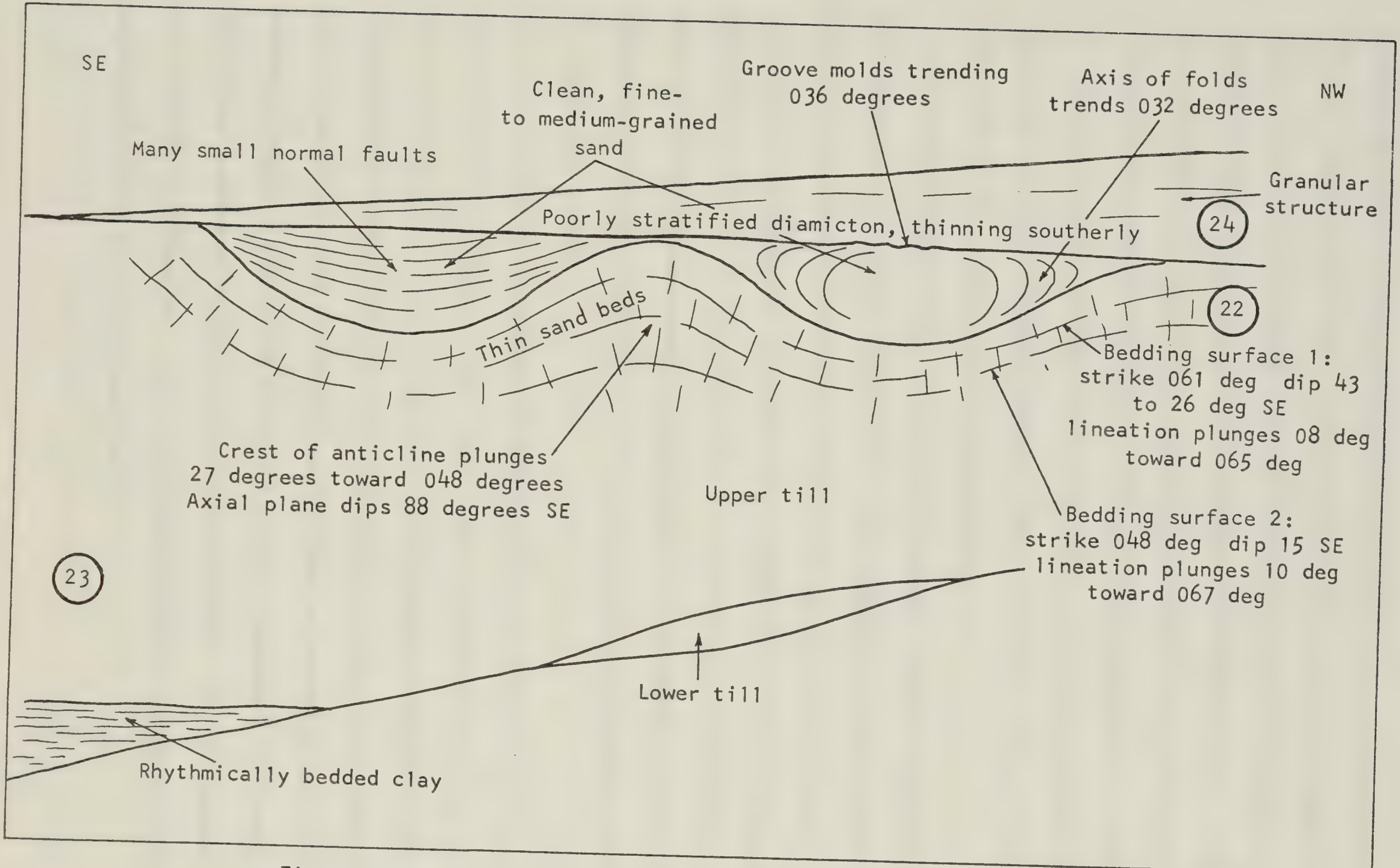


Figure 22. Sketch (not to scale) of exposure at Location K showing structural features.

dipping 50 degrees toward 359 degrees and a preferred orientation of 068 23. Sample 23 is from a few feet above the base of the upper till. Comparison with test samples 505-507 (fig. 5) reveals no reason for inferring other than a random distribution of the pebble axes. Sample 24 is from the uppermost diamicton above sample 22. It clearly shows a relatively strong preference for northerly to northeasterly trends. In view of the high concentrations of observations in this region of the projection, it is thought probable that the bimodal nature of the sample reflects a bimodality in the population.

The near parallelism of the fabric maximum from the upper part of the upper till to the lineations on the bedding surfaces suggests that both the fabric and the lineations were produced by the same movement. This movement could have been either a series of subaqueous gravity mass flows (density currents) in a lacustrine environment, or shearing due to ice movement in a sub-glacial environment. In the latter case, the sand beds could perhaps be explained by the "undermelt theory" of Carruthers (1953).

The origin of the uppermost diamicton is not clear. Its stratification, granular structure and rapid thinning southward suggest a local gravity flow. Such a flow could have produced the grooves on the base of this unit. However, the truncation of the folded upper till with its overlying sand (although possibly due to erosion by density currents) suggests overriding by a late glacial readvance. Evidence of such a late readvance is seen in the form of moraines in the Redwater area northeast of Edmonton. It is not known at this time whether this readvance extended as far as Location K.

Location L

Location L is a large gravel pit where numerous exposures of both tills have been made. No sediments intervene between the tills. Four fabric samples are available from the upper till and three from the lower till as indicated roughly by the sketch in Figure 13.

The lower till contains three structurally distinct vertical zones. At its base is an intensely sheared, continuous, black clayey till layer one to two inches thick. Plate 1-B shows the basal portion of a till block collected from the base of the lower till at site 45. The contact with the underlying Saskatchewan Gravels can be seen, and the basal sheared zone is clearly discernible. At one point within this layer, near-horizontal slickensides were observed, having a trend of 015 degrees.

The middle zone is dense and massive, and has a columnar structure (Plate 1-D, E). Its thickness varies from zero to about four feet. Samples 45 and 466-10 (Fig. 13) are from this zone. Both show a near-horizontal girdle and a preference for northwest trends with fairly high horizontal scatter.

The upper zone of the lower till is highly fractured and contains many shear surfaces (Plate 1-D, E). Lineations (Plate 1-C) and grooves (Plate 1-D) on near-horizontal shear surfaces have trends between 020 and 025 degrees. Sample 43 is from this zone. Preference for north-northwesterly and northeasterly trends is apparent. The two groupings in the sample probably represent two preferred orientations in the population, since the high concentration of observations in this region affords a better than usual degree of reliability. The two groupings

are centred at approximately 346 35 and 016 10.

Sample 466-1 from the upper till shows two well-defined preferred orientations: one at 340 16 and one at 031 10. Sample 466-3 shows a preference for trends in the northerly and southerly quadrants between northwest-southeast and northeast-southwest. The maximum at 213 06 may represent a preferred orientation. Sample 466-8 indicates a preferred orientation of 209 11 and a preference for a near-horizontal plane. Sample 466-11 apparently indicates two preferred orientations at 010 06 and 075 38.

Location M

At this gravel pit only one till, the upper till, was present in the exposure mapped. Sample 466-9 (fig. 7) shows a preference for a plane dipping 06 degrees toward 310 degrees, but no reliable indication of a preferred trend.

Upper Till Fabrics and Glacier Movement

Well developed groove molds on the base of the upper till in the Clover Bar area just east of Edmonton (Plates 1-A, 2-E) trend from 028 to 035 degrees and indicate that the sense of movement was from northeast to southwest (Westgate, 1968). Such a line of movement for the glacier that deposited the upper till is also indicated by flutings (Bayrock and Hughes, 1962).

The preferred trends of elongate pebbles in the upper till, inferred from the present study as discussed in a previous section, are

indicated in Figure 23. To these have been added sample AFL (Lorberg, 1967) and sample RBR-1b (Rains, 1969) measured by other investigators.

(Rains (1969, p.18) tested the aximuth distribution of sample RBR-1b, using 18 ten-degree classes, against uniformity using chi-square and found no significant deviation. However, while a high value of chi-square can justify rejection of the hypothesis model, a low value of chi-square does not necessarily justify acceptance of the model. The low chi-square value calculated by Rains does not justify acceptance of the model of uniformity. The writer tested the same sample using a different set of classes and obtained a chi-square value that did justify rejection of the model of uniformity.)

Of the seven samples from the upper till at Location F, five show well-defined preferred trends. These five preferred trends range from 000 degrees to 066 degrees (fig. 12). The four samples from the upper till at Location L show six preferred trends varying from 340 degrees to 075 degrees (fig. 13). One sample is available from the upper till at each of the other locations shown in Figure 23; except for those at Locations G, I, and J, these show preferred trends ranging from 008 degrees to 066 degrees. It cannot be assumed that the degree of fabric variability seen at Locations F and L does not exist at these other locations. In the absence of information about the variability of the fabric at these locations, the preferred trends observed in single samples cannot be regarded as accurate indications of ice-movement directions.

All the preferred trends shown in Figure 23 lie in the range 340 degrees to 075 degrees, except for samples 33 (Location B, fig. 10), 32 and 466-5 (Location G, fig. 7), 17 (Location I, fig. 9) and 20 (Loca-

Figure 23 Map showing preferred trends of elongate pebbles in the upper till.

tion J, fig. 9). The preferred trends shown by these five samples are all west-northwest-east-southeast to northwest-southeast, that is, approximately normal to the direction of movement of the glacier as indicated by sole markings and surface features. Thus they may represent pebbles oriented transverse to the movement direction. All five samples show a high degree of horizontal scatter; it is possible that this is characteristic of transverse fabrics.

Striations on a shear surface within the clayey layer at the base of the lower till at Location F indicate two principal lines of movement: 033 ± 6 degrees and 000 ± 2 degrees. It will be shown in the later discussion of the lower till that these striations are probably related to movement of the glacier that deposited the upper till. The inference of two principal directions of movement for this glacier is supported by the orientations of the more elongate pebbles from the upper till at Location F (fig. 14 and 15), which show preference for the positions 022 07 and 349 22. Moreover, as pointed out previously, as the elongation of the pebbles increases (fig. 14, top row) the grouping at 349 19 increases in strength while that at 029 15 becomes weaker and finally disappears. If it is supposed that the more elongate pebbles will be the first to become re-aligned to a new movement direction, then the above observation indicates that movement from about 349 degrees occurred later than movement from about 022 degrees to 029 degrees. This is consistent with the striations described above. The weakening of the grouping at about 055 27 with increase in elongation of the pebbles may be regarded as suggesting that the movement from 022 degrees to 029 degrees was preceded by still earlier movement from about 055 degrees.

Figure 15B shows the distribution of c-axes of flat pebbles, again

from the upper till at Location F. The majority of these c-axes are nearly vertical, confirming the observations of Harrison (1957a) and Johansson (1968). A second group of c-axes is seen to trend southeasterly to easterly with plunges from 34 degrees to 18 degrees, indicating a group of flat pebbles with their a:b planes striking from northeast to north and dipping steeply to the northwest or west. If it is supposed that the steeply dipping a:b planes of such flat pebbles will tend to assume an orientation such that they strike nearly parallel to the direction of ice-movement, then the observed northeasterly to northerly strikes are consistent with ice movement ranging from northeast-southwest to north-south. Presumably the a:b planes initially assumed a near-vertical position with their strikes parallel to the early movement direction from northeast to southwest. It is suggested that as the movement direction shifted toward a north-south line, some of these a:b planes became re-aligned to strike parallel to the new movement direction; those that did not become re-aligned were tilted away from their vertical attitude as a result of shearing or flow of the till. The longer the period of movement oblique or transverse to the original strike of an a:b plane, the more it would tend to be tilted if it did not become re-aligned to strike parallel to the new movement direction. Thus, a:b planes with the "oldest" strikes would be expected to have the greatest deviation from the vertical, or the smallest dips. Such a relationship is seen in Figure 15B if the suggested history of ice movement is correct: the a:b planes stiking northeasterly (parallel to earlier movement) dip less steeply than the a:b planes striking northerly (parallel to later movement).

One possible explanation of the apparent change in ice movement

direction from northeast-southwest to approximately north-south at Location F is the increasing influence of local topography on the glacier as it thinned. The bedrock valley trending north-south about one mile west of Location F (fig. 17) may have been responsible for channeling the thin waning glacier into a north-south flow pattern. Such late-stage changes in flow direction due to increasing influence of local topography would clearly be different in different parts of the study area. Johansson (1968, p.210) believed that control of ice movement by local topography was related to the existence of a relatively thin ice cover.

The wide spread of preferred trends seen in the upper till in the Clover Bar area may thus be due to the wide range of movement directions to which this area may have been subjected as a result of the local topography, and may not be found at other localities.

Lower Till Fabrics and Glacier Movement

The preferred trends shown by samples believed to be from basal lower till are shown in Figure 24. Fabrics of units thought to be turbidites were excluded. Sample 8 was also excluded; its steeply dipping girdle, being uncharacteristic of till fabrics, suggests that the granular-structured "phase" of the lower till in which it was measured is an inclusion of previously deposited sediment retaining its original fabric but having been rotated. Before attempting to infer any ice movement directions, the first fact that must be considered is that the pebble fabric of the lower till is thought to have been altered by the overriding later glacier at several of the

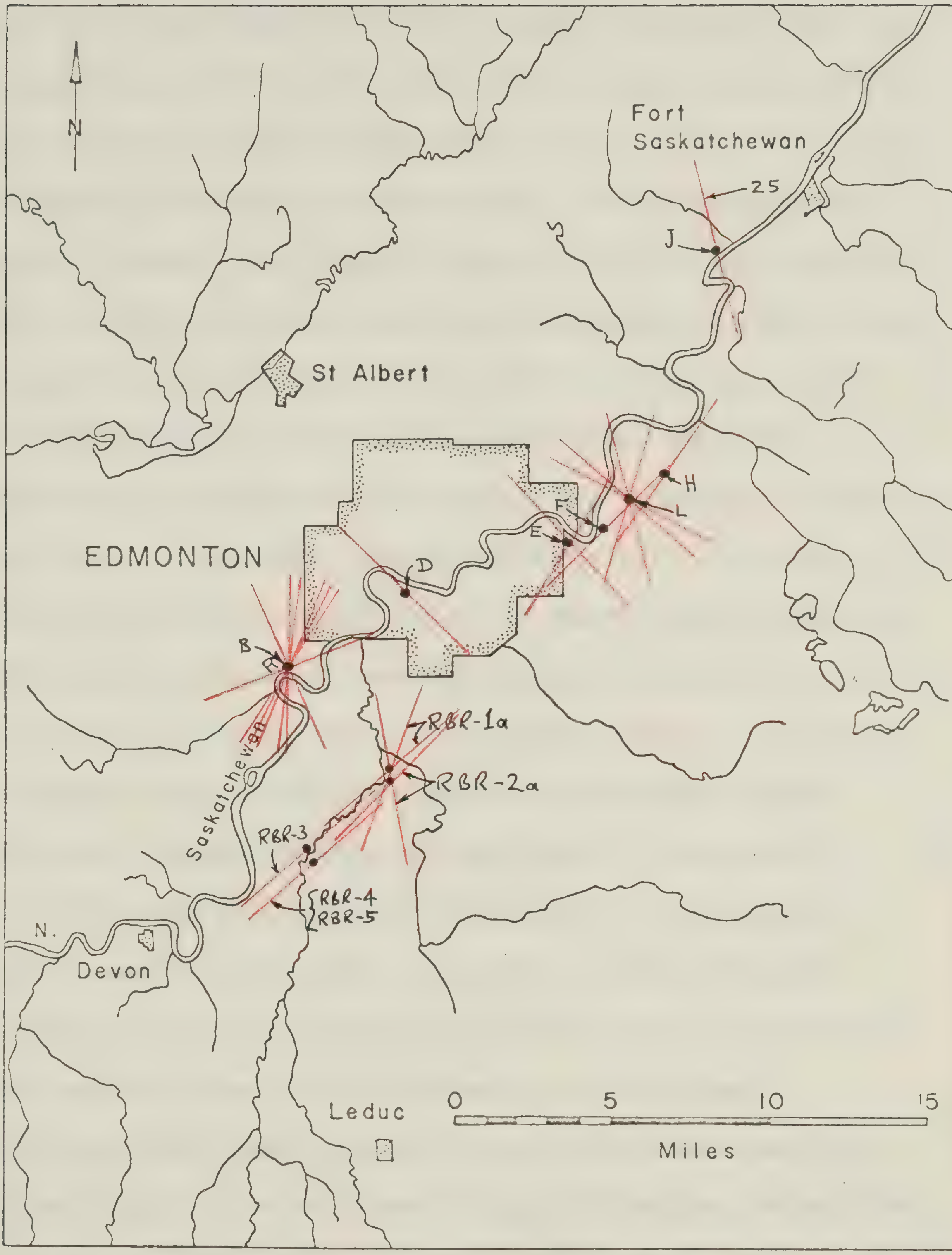


Figure 24 Map showing preferred trends of elongate pebbles in the lower till.

locations studied. The evidence for such alteration at Location L has been described in detail elsewhere (Ramsden and Westgate, in press) and is briefly as follows. The fabric (fig. 13, samples 45, 466-10) of the middle zone of the lower till (Plate 1-B,D,E) is thought to be the original depositional fabric of the lower till for two reasons: (1) it consists of a near-horizontal girdle with a preferred direction in this girdle, a pattern described by several workers from till in areas of ground moraine (Holmes, 1941, p.1313; Harrison, 1957a, p.283; Kauranne, 1960, p.87), and (2) the middle zone is much less sheared and fractured than the upper zone, it is comparatively massive and dense, and its columnar structure resembles that of the undisturbed upper till.

Both structural and fabric observations are important in interpreting the fabric of the upper zone of the lower till. The near-parallelism of the slickensides in the basal sheared zone of the lower till (with trends of between 015 and 035 degrees) to the grooves and lineations in the upper zone (Plate 1-C,D) (with trends of 020 to 025 degrees) suggests that all these similarly oriented features were produced at about the same time by the same agent of deformation. Furthermore, the close parallelism of the trends of these features to the known direction of movement of the later glacier (fig. 13, top row; Plate 1-A; Plate 2-E) strongly suggests that the movement of this glacier was the cause of the deformation in both zones.

The grooves and lineations in the upper zone of the lower till are also nearly parallel to the trend of about 016 degrees of the fabric maximum of this zone (fig. 13, sample 43). This suggests that shearing altered the original fabric of the till, producing the present configuration. The other maximum in the fabric of this upper zone may

represent a remnant of the northwesterly trending maximum of the original fabric.

Other fabric samples from the lower till are thought to represent fabrics partially altered by the overriding later glacier. Sample 13 (fig. 6), from the top of the lower till at Location D, has a secondary maximum at 021 00 that may be due to overriding by the later glacier. Since the diamictons that overlie the lower till at this location are thought to be lacustrine turbidites, it cannot be verified that the later glacier actually moved from 021 degrees at this place; however, it is not unlikely in view of the upper till fabrics in the area (fig. 23). Sample 25 (fig. 9) from the top of the lower till at Location J has a maximum at 346 41 and a secondary maximum at 206 11, and may represent another partially altered fabric. If the preferred trend of about 306 degrees in sample 20 from the upper till at this location represents pebbles transverse to ice movement as it is thought, then it is consistent with the postulated partial alteration of the lower till fabric.

In addition to the above-mentioned fabrics that are thought to be partially altered, two of the samples from the lower till are thought to represent fabrics completely altered by the overriding later glacier. This inference is made on the basis of parallelism or near-parallelism of the fabric maximum to the known direction of advance of the later glacier, combined with intense fracturing and shearing of the lower till parallel to this direction of advance. Such fabrics are samples 7 and 21 from the lower till at Location F (fig. 6; Plate 2-C). Both of these fabrics have a near-horizontal girdle and a preference for trends of about 045 degrees. Sample 7 shows a transverse peak. Thus these altered fabrics show the same characteristics as primary till

fabrics. Other samples taken from highly fractured lower till show maxima with trends within the known range of movement directions of the later glacier, but where shearing of the lower till parallel to this direction could not be demonstrated beyond doubt the origin of the lower till fabric cannot be determined. Indeed, perhaps the biggest single problem in interpreting lower till fabrics in the Edmonton area is that of differentiating between primary fabrics and those produced by the overriding later glacier.

The preferred trends of lower till fabrics were re-plotted in Figure 25, omitting those described above as wholly or partially altered by the later glacier. The remaining preferred trends at Locations H, L, E, and D, those on samples RBR-3, RBR-4, and RBR-5, and the more easterly maxima on samples RBR-1a and RBR-2a all lie in the two groups 118 degrees to 138 degrees and 032 degrees to 050 degrees. These two intervals are centred at approximately 130 degrees and 040 degrees, that is, are close to 90 degrees apart. It is possible that these two directions are parallel and transverse to the dominant movement direction of the glacier that deposited the lower till. The maxima in samples RBR-1a and RBR-2a that do not fit the northeasterly trending group could have been produced by the later glacier, although no structural information is available from these sites at this time.

The wide range of preferred trends at Location B may be due to changes in movement direction resulting from varying degrees of influence of the local topography as the thickness of the ice-sheet changed. Situated on a major bedrock valley close to its confluence with other major valleys, the location would be susceptible to such changes.

Only two of the samples whose preferred trends lie in the two

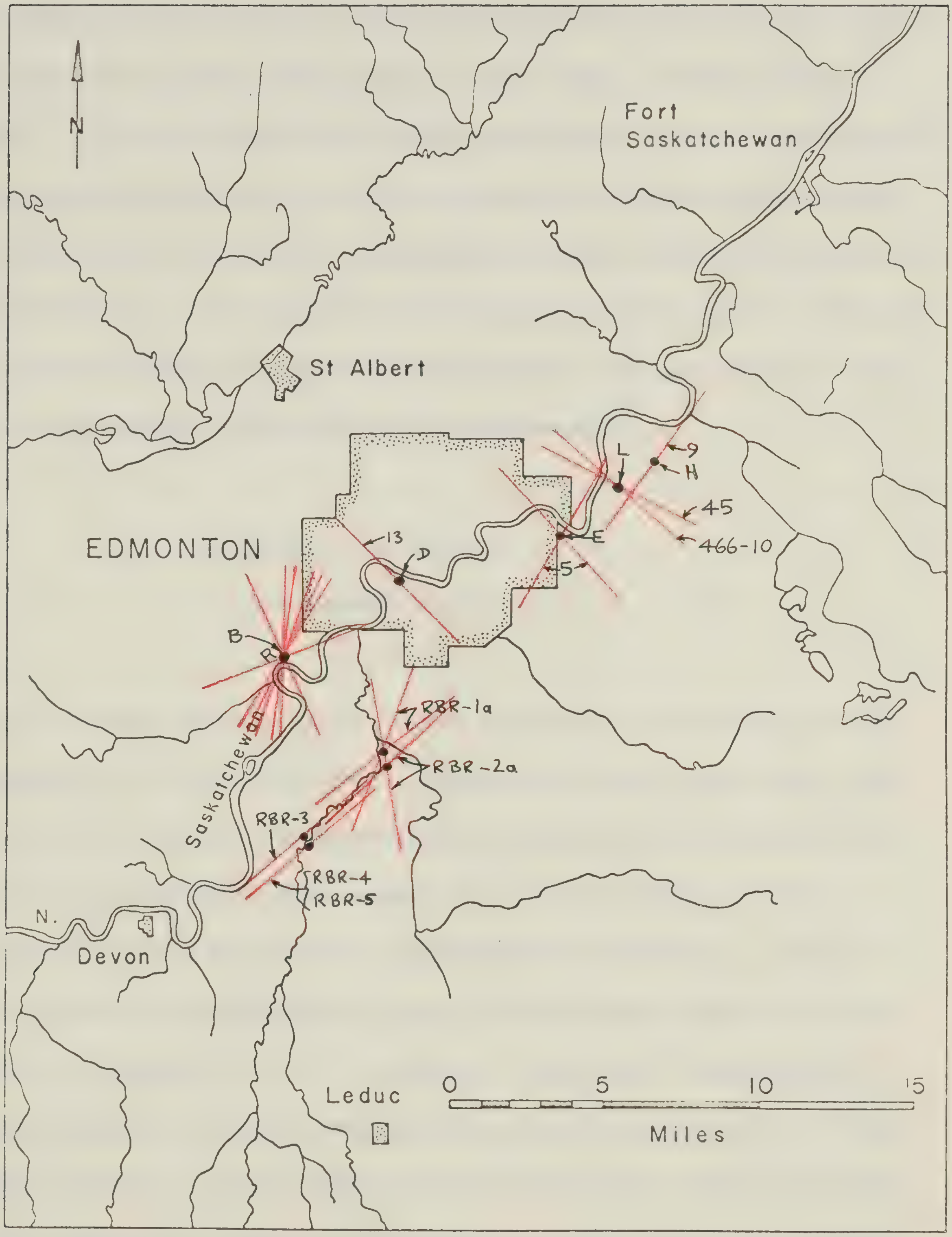


Figure 25 Map showing preferred trends of elongate pebbles in the lower till, with wholly or partially altered fabrics omitted.

groups described above are accompanied by any positive evidence that they represent primary lower till fabrics. These are samples 45 and 466-10, both from Location L (fig. 13), and the evidence is mainly the contrast between the structural characteristics of the middle zone of the lower till from which these samples where taken and the sheared upper zone. It may be worth noting that both these samples show a well-defined near-horizontal girdle but a high degree of horizontal scatter about the preferred trend. As mentioned previously, the five upper till fabrics that have a similar pattern are thought to be transverse fabrics, suggesting that the glacier that produced fabrics 45 and 466-10 in the lower till was moving from northeast to southwest.

Value of Fabrics as a Stratigraphic Tool

It can be seen from Figures 23 and 24 that no preferred trend is found exclusively in one till unit. Since both tills show a very wide range of preferred trends, determination of the preferred trend of the pebbles in an unidentified till cannot serve to identify the till. Three samples were taken from the Lake Edmonton diamictons -- two at Location H, several feet above the lower limit of observable stratification (fig. 7, samples 10, 11), and one at Location F about one foot above the lowermost visible stratification (fig. 6, sample 37). These three fabric patterns show no apparent characteristics that would allow them to be singled out as being different from the rest of the samples. However, groups 37-1 and 37-2 of sample 37 (fig. 11) indicate a substantial change in the fabric over a distance of less than a foot; the sequential subgroups of samples 34 and 35 from the upper till at the

same exposure show no such change. Thus this extreme variability may be characteristic of the lake sediments and perhaps of lacustrine diamictons generally, but no other data are available to illuminate this problem further.

Two samples were taken from units regarded as possible turbidites on the basis of structural and (or) stratigraphic evidence. These samples are 1 (Loc. D, fig. 6) and 8 (Loc. H, fig. 7). Five more samples from two more locations are from units regarded as probable turbidites; these are 18, 26 and 44 from Location I and 22 and 24 from Location K (fig. 9). These samples all show a relatively strong preferred orientation, lack a continuous girdle (except sample 8), and lack transverse maxima (except sample 1). To test the possibility that these features are characteristic of turbidite fabrics, the same features were looked for on other samples believed to be from till. Other samples with these features are 466-4 and 466-2 (Loc. B, fig. 10), 6 (Loc. F, fig. 12), and 466-1 and 466-8 (Loc. L, fig. 13).

Sample 466-4 is from the top of the upper till immediately below Lake Edmonton sediments. It could conceivably represent a lacustrine turbidite. Sample 466-2 is from the basal part of the lower till. This basal portion of the lower till is partially stratified, contains thin sand beds and is quite possibly lacustrine in origin. Sample 6 is from the middle of the upper till which is not likely to be a turbidite. Sample 466-8 was taken from well below the top of the upper till and is also almost certainly a till fabric. The stratigraphic position within the upper till of sample 466-1 is uncertain, and it is possible that it came from basal Lake Edmonton sediments. Clearly the available information allows no conclusions to be drawn about differentiation of tills and

turbidites by means of their pebble fabric.

Lindsay (1968) simulated "mudflow" fabrics using a computer as well as studying real "mudflow" and tillite fabrics. He concluded that whereas the tillite fabrics had only a single mode, mudflow fabrics were characterized by a dipping girdle, sometimes with a maximum on this girdle. This contrasts with the results of other studies, including the present one, that indicate a near-horizontal or moderately dipping girdle, frequently with a maximum on it, is the characteristic pattern of till fabrics in areas of ground moraine. Lindsay, et al (1970) noted that all "mudflow" fabrics examined by them had single modes paralleling the flow direction, some with and some without a girdle. They thus lacked transverse modes. All the fabrics of probable turbidites listed above lack transverse modes. Thus the transverse mode may be peculiar to till fabrics and normally lacking in mudflow or turbidite fabrics.

Glen, Donner and West (1957, p.194) pointed out that "Laboratory experiments and theoretical studies by various authors have shown that an initially random collection of elongated objects immersed in a flowing liquid will develop a long axis distribution with a peak parallel to the direction of flow in a very short time, but that if flow is continued for a long time a transverse peak will also develop." If this is true, it would not be surprising to find that turbidite and mudflow fabrics lacked the transverse mode, since their period of formation is very much shorter than for till fabrics, and the distance of transportation probably much smaller, so that they would be much less well developed than till fabrics.

The fabrics of stratified Lake Edmonton diamictons (samples 10 and 11, Loc. H, fig. 7; sample 37, Loc. F, fig. 6) are not similar to

the fabrics of the probable turbidites. All three have a near-horizontal girdle; sample 10 is very similar to sample 5 (Loc. E, fig. 6), a probable till fabric. Sample 11 shows a preference for northeast-southwest and east-west trends. Sample 37 probably represents a uniform girdle.

Microfabric studies might be able to yield far more information than pebble fabric studies for the same investment of time, and may prove to be a more powerful tool than pebble fabrics in the determination of sediment origin.

Work Needed

More fabric and other stratigraphic data are needed before the direction of movement of the earlier glacier can be determined. The effect of local topography on ice-movement directions in the Edmonton area could be better assessed if more fabric data were available at locations away from the main bedrock valleys. Differentiation of primary fabrics and fabrics altered by shearing resulting from overriding by a later glacier could perhaps be achieved through study of the micro-structural features of the sediment. Further work is required on the fabrics of diamictons of different origins, for example, tills, turbidites, and ice-rafted deposits.

CONCLUSIONS

A method of statistically treating till pebble orientations based on an assumed probability distribution has been used by several workers in recent years. The method was proposed by Andrews and Shimizu (1966) and based on the assumption that the distribution of till pebble axes may be described by a probability density function due to Fisher (1953). The method was examined and found to be unsatisfactory for the following reasons: (1) The condition $k \geq 3$, where k is Fisher's precision parameter, is used as the sole criterion of applicability of the method, that is, as indicating that the distribution is essentially unimodal. This is not true. (2) The recommended procedure for deleting transverse pebbles fails. (3) Watson's (1956b) test for randomness of directions, based on the length of the resultant vector, is not valid for hemispherical distributions.

The failure of the method, adopted from a procedure designed to treat paleomagnetic vectors, is attributed largely to the fundamental differences between magnetic vectors and pebble axes. Thus a thorough investigation of the differences between different kinds of orientation data and of the distinct functions of reference planes used in their treatment must precede the development of suitable methods of treatment.

An alternative approach, assuming Fisher's probability distribution for each mode of a multi-modal pebble axis distribution, was also examined. Testing of till fabric data, however, showed that in the majority of cases even the individual modes did not conform to Fisher's density function.

The calculation of descriptive statistics for till fabric samples

based on valid three-dimensional models must await further work on either (1) models for elongate groupings, or (2) procedures permitting a grouping to be treated separately from a girdle occurring in the same sample. Until adequate techniques for the numerical representation of till fabric data are devised and widely accepted, till fabrics should be represented graphically to avoid possible loss of information.

Knowledge of the distribution of R, the length of the resultant of a set of unit vectors, for a hemispherical distribution would be a valuable aid in the interpretation of till fabric samples.

Fifty pebbles with a-axes of at least one inch and a:b ratios of at least 1.2 are barely adequate to define the fabric of a till at a single point. Preferred orientations are indicated only approximately.

Although different samples from a till unit at a given exposure are in some cases very similar, this can by no means be counted upon, since frequently great variation is evident. It must be concluded that a single one-point sample should never be depended upon to indicate the preferred trend of elongate pebbles in a till unit at an exposure. Indeed, the assumption that the pebbles in a till at an exposure have a single preferred trend must be questioned. They may have many preferred trends, or different preferred trends in different parts of the exposure, both laterally and vertically.

A sample of less than about 200 long axes from the nine sample sites in the upper till at Location F could not be relied upon to indicate the northeasterly trending preferred orientation. This is attributed to the degree of fabric variability of this unit at this location.

More elongate pebbles, that is, pebbles with higher a:b ratios, appear to be more sensitive indicators of ice-movement direction. They are apparently the first to become re-aligned with a new direction of ice movement, so that they may be found to prefer a later direction of movement while less elongate pebbles still prefer an earlier direction.

No relationship was found between the size of a pebble and its preferred orientation. The size was, however, found to have considerable influence on the tightness of grouping of pebbles about their preferred orientation. Harrison's (1957a) conclusion that the majority of flat pebbles have their c-axes nearly vertical was confirmed. The imbrication of these pebbles, if any, could not be determined.

Five fabric samples from the upper till are thought to represent transverse fabrics. They all show little vertical scatter but much horizontal scatter. This may be characteristic of transverse fabrics. The direction of movement of the glacier that deposited the upper till at Location F appears to have changed from northeast-southwest to north-south during the later part of the glaciation. This change may be due to the increasing influence as the glacier thinned of the north-south-trending bedrock valley about one mile west of this location (fig. 17). The wide spread of preferred trends seen in the upper till in the Clover Bar area may thus be due to the wide range of movement directions to which this area may have been subjected as a result of the local topography, and may not be found at other localities.

The fabric of the lower till has been wholly or partially altered by the overriding later glacier at several of the locations studied. One

of the major problems in interpreting lower till fabrics in the Edmonton area is that of differentiating between primary fabrics and those produced by the later overriding glacier. Insufficient data are available to determine the direction of movement of the glacier that deposited the lower till.

Fabric samples from stratified Lake Edmonton diamictons showed no features that could serve to distinguish them from till fabrics. Several samples from probable lacustrine turbidites, however, suggest that fabric data might help to distinguish such deposits from tills.

More information is required concerning fabric variability, the effect of pebble shape on pebble orientation, and the effect of local topography on ice movement.

The potential value of studies of micro-fabrics and micro-structural features of tills and other diamictons should be investigated.

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APPENDIX A

PLATES

PLATE I

A Groove molds trending 028 to 033 degrees at the base of the upper till at Location M. The undercut is about two and a half feet deep.

B Till block collected from the base of the lower till at Location L. The basal sheared zone is clearly visible.

C Lineations trending 020 to 025 degrees on a near-horizontal shear surface in the upper zone of the lower till (see text). The coin is a dime.

D Undulating shear surface separating the upper zone (b2) from the middle zone (bl) of the lower till at Location L. The undulations are approximately parallel to the movement direction of the glacier that deposited the upper till (approximately 025 to 035 degrees). Photo covers about eight feet vertically at centre. Letters and numbers have same meaning as in E.

E The section exposed at Location L: a. Saskatchewan Gravels. bl. Middle zone of lower till. b2. Upper zone of lower till. c. Upper till. d. Lake Edmonton sediments. The prominent shear surface separating the middle and upper zones of the lower till is indicated by dashed lines. The pick is 17 inches long.

Plate I

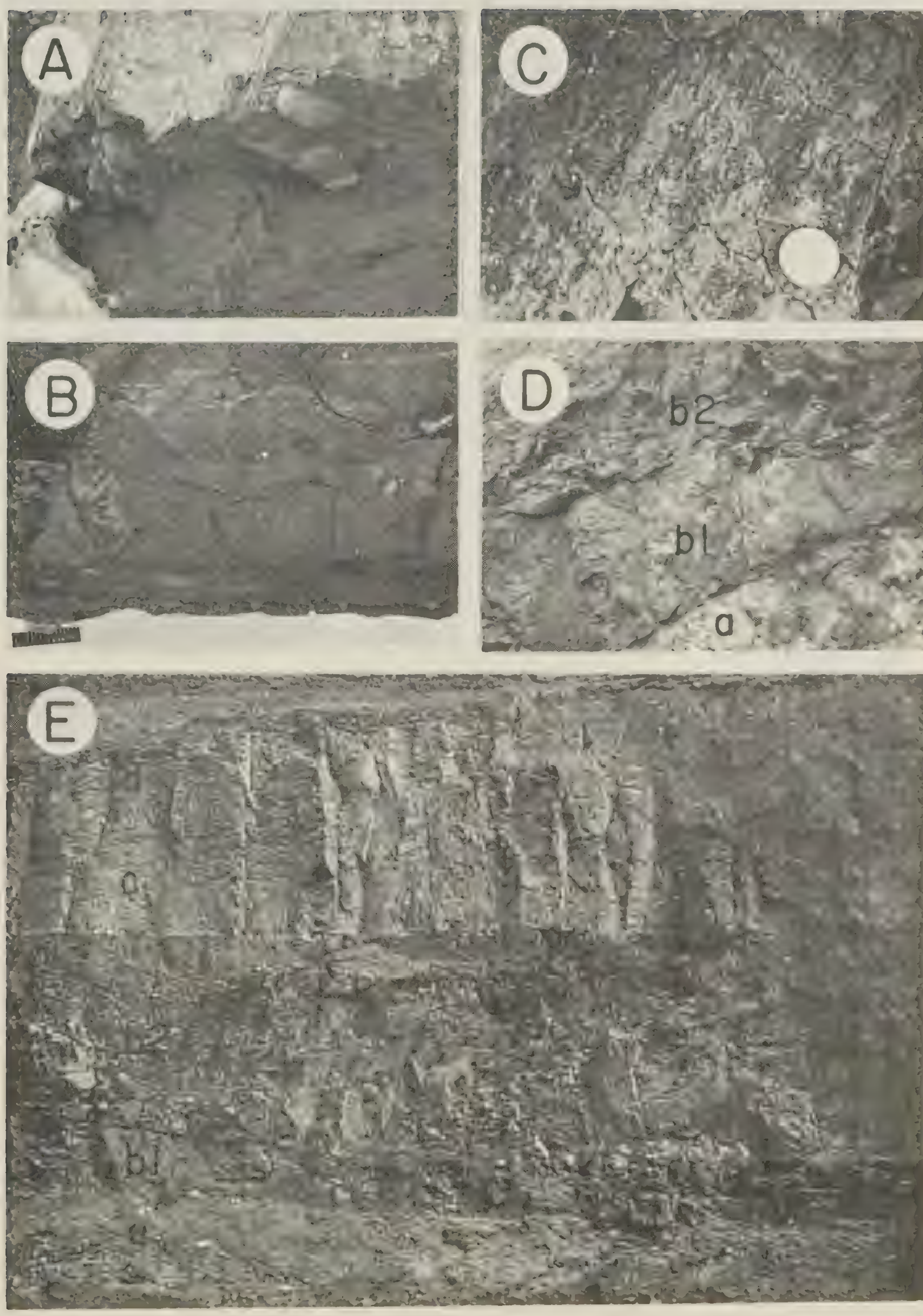


PLATE 2

A, B Stereopair showing the basal sheared zone of the lower till at Location F. The prominent flat shear surface on which striations were measured can be seen about one inch above the top of the underlying sand of the Saskatchewan Gravels. Vertical separation of the two pebbles is about three inches.

C Part of the lower till at Location F showing its highly fractured character. The two pebbles that appear in A and B can be seen to the left of the mattock head. The mattock is 17 inches long.

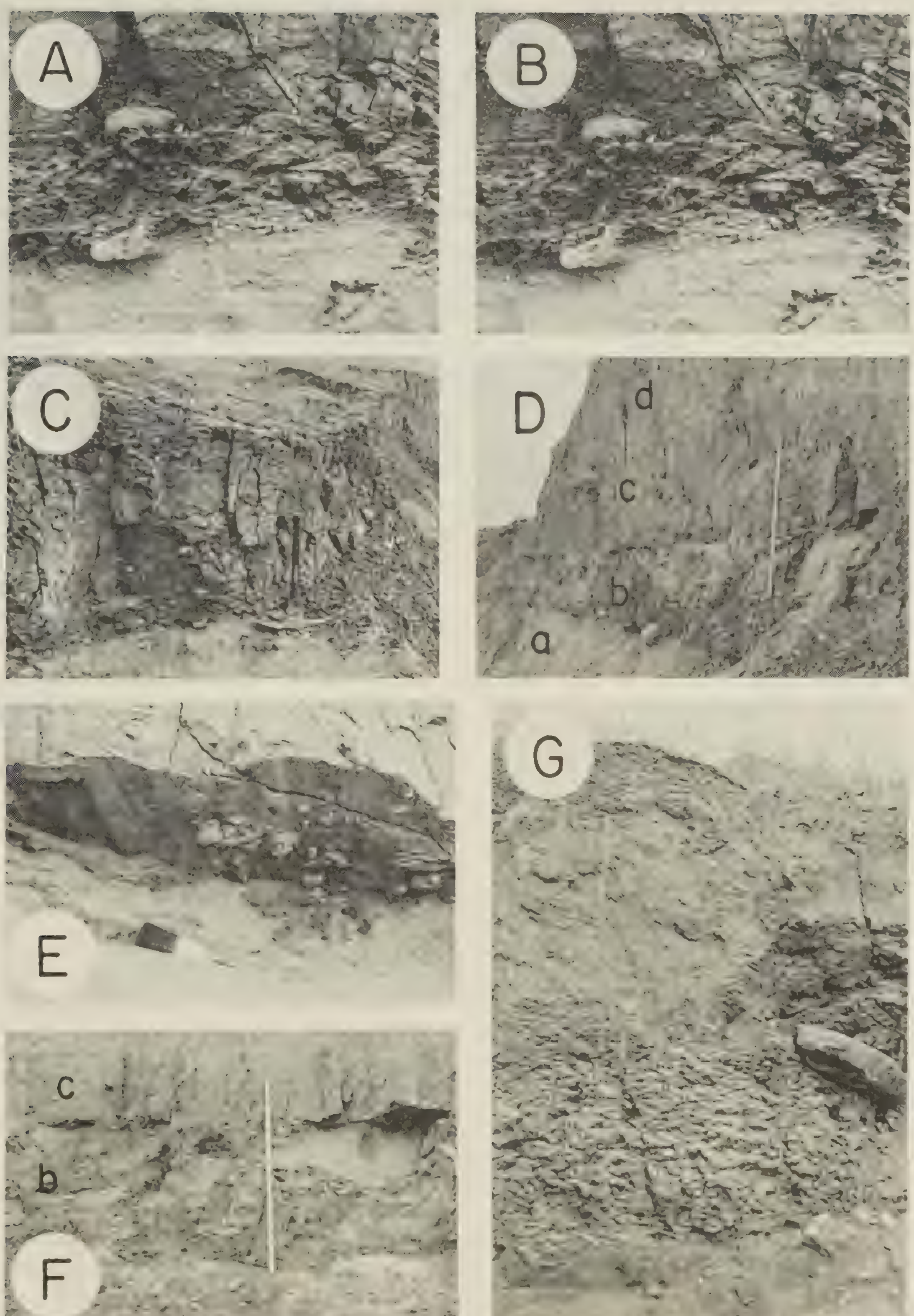
D The section exposed at Location H: a. Saskatchewan Gravels. b. Lower till. c. Upper till. d. Lake Edmonton sediments. The rod is four feet long.

E Groove molds trending 035 degrees at the base of the upper till at Location H. The brush is about eight inches long.

F The lower till and base of the upper till at Location H, showing inclusions of clean, fine- to medium-grained sand in the lower till. The fractured nature of the lower till (b) contrasts with the columnar jointing of the upper till (c). The rod is four feet long.

G The lower till at Location H, showing normal dark gray, blocky lower till below and the brown, granular-textured "phase", believed to be an inclusion of previously deposited sediment, above. Diameter of large stone is about one foot.

Plate 2



APPENDIX B

MEASURED SECTIONS

Location A

East bank of North Saskatchewan River, SW¼, sec.6, Tp.51, R.25, W4. Stratigraphy from unpublished report by G. Gabert, Geology Department, University of Alberta, 1967.

Description	Average Thickness (ft)
Pleistocene	
Silts and clays, bedded	34
Till, brown, sandy	2
Silt and sand, bedded	12
Till, clayey, dark grey	3
Saskatchewan Gravels: sand	49
Cretaceous	
Edmonton Formation	4+

Location B

North bank of North Saskatchewan River; middle of west side of sec.15, Tp.52, R.25, W4.

Description	Average Thickness (ft)
Pleistocene	
Clay and silt, bedded	8
Till, brown vertical joints	10
Till, grey-brown to dark grey	45
Fine sand and silt, current-bedded	2
Sand, medium-grained, buff, stratified	34
Saskatchewan Gravels: sand and gravel	11
Cretaceous	
Edmonton Formation	2+

Location C

East bank of North Saskatchewan River, middle of west side of sec.14, Tp.52, R.25, W4. Stratigraphy from unpublished report by E F Lorberg, Geology Department, University of Alberta, 1967.

Description	Average Thickness (ft)
<u>Pleistocene</u>	
Clays and silts, bedded	27
Till, dark grey, lighter colours towards top, columnar jointing	18
Saskatchewan Gravels: sand and gravel	66
<u>Cretaceous</u>	
Edmonton Formation	2+

Location D

Excavation for new Biological Sciences building, north end of University of Alberta campus; middle of south side of sec.31, Tp.52, R.24, W4.

Description	Average Thickness (ft)
Pleistocene	
Clay, massive, brown, scattered pebbles	10.0
Clays, silts and fine sands, brown and grey, highly contorted	2.1
Silt and fine sand, buff to brown	1.6
Diamicton, brown, many vertical joints	2.0
Silt, fine sand, brown to buff, lenses of brown, sandy diamicton	4.0
Diamicton, brown	1.0
Silt and sand	1.0
Diamicton, brown	2.0
Sand, medium-grained, clean, buff	2.0
Silt and fine sand, brown	4.0
Sand, fine- to medium-grained	4.0
Till, clayey, dark grey	2.0+

Location E

Abandoned gravel pit just east of creek, sec.1, Tp.53, R.24, W4.

Description	Average Thickness (ft)
<u>Pleistocene</u>	
Silt, clay, fine sand, buff to brown, scattered pebbles, highly contorted	8
Till, sandy, brown, medium blocky structure to massive. Inclusions of buff to brown silt and sand up to at least 4 ft x 20 ft. Includes large mass (larger than 5 ft x 20 ft) of Cretaceous bedrock.	26
Till, clayey, dark grey, fine blocky structure	12 approx.
Saskatchewan Gravels: sand	6+

Location F

Twin Bridges Sand and Gravel Co. Pit No. 2. NE¼, sec.6, Tp.53, R.23, W4.

Description	Average Thickness (ft)
<u>Pleistocene</u>	
Silt and clay, stratified, brown to buff. Scattered pebbles and less stratification in lower part	10
Till, brown, columnar jointing	6
Till, dark grey, fine to medium blocky structure	10
Saskatchewan Gravels: sand and gravel	30 approx.

Location G

Gravel pit owned and operated by G. Kropp in north central part of sec.8, Tp.53, R.23, W4.

Description	Average Thickness (ft)
<u>Pleistocene</u>	
Sand and clay, stratified	7
Till, sandy, buff to brown. Columnar jointing	14
Till, clayey, grey. Blocky structure.	1
Saskatchewan Gravels: sand and gravel	20+

Location H

Twin Bridges Sand and Gravel Co. Pit No. 1. SW¼, sec.16, Tp.53, R.23, W4.

Description	Average Thickness (ft)
Pleistocene	
Clay and silt, stratified, scattered pebbles. Less well stratified and more pebbly in lower part.	10
Till, brown, columnar jointing	3
Till, dark grey, fine to medium blocky structure. Inclusions of clean, medium- to coarse-grained sand in upper part. Also inclusion of brown diamicton with granular structure.	3
Saskatchewan Gravels: sand	6+

Location I

East bank of North Saskatchewan River, middle of east side of sec.34, Tp.53, R.23, W4.

Description	Average Thickness (ft)
Pleistocene	
Silt and clay, stratified	11.3
Till, light brown, columnar jointing	10.9
Diamicton, dark grey, columnar jointing	0.9
Sand, silt and clay, stratified, includes 4-inch bed of brown diamicton	3.6
Diamicton, dark grey, columnar jointing	2.0
Sand, silt and clay, stratified	2.0
Diamicton, dark grey, granular structure	3.2
Clay, dark grey, granular structure	3.4
Saskatchewan Gravels: sand and gravel	33 approx.
Cretaceous	
Edmonton Formation	25+

Location J

North bank of North Saskatchewan River, middle of north side of sec.23, Tp.54, R.23, W4.

Description	Average Thickness (ft)
Pleistocene	
Diamicton, stratified	8.7
Silt	2.0
Till, brown, columnar jointing	2.0
Sand	0.9
Till, grey, blocky structure, sand and gravel inclusions	15 approx.
Cretaceous	
Edmonton Formation	3+

Location K

Gulley formed by surface runoff, on Pointe-aux-Pins Creek in extreme SW corner of sec.32, Tp.53, R.22, W4.

Description	Average Thickness (ft)
Pleistocene	
Diamicton, brown, stratified, granular structure	2
Till, brown, beds of sand in upper part	10 approx.
Till, dark grey, massive	1
Cretaceous	
Bearspaw Formation	2+

Location L

Gravel pit operated by Alberta Concrete Products Limited. NW¼, sec.8, Tp.53, R.23, W4.

Description	Average Thickness (ft)
Pleistocene	
Silt and clay, stratified	9
Till, brown	10
Till, dark grey	7
Saskatchewan Gravels: sand and gravel	10+

Location M

Gravel pit operated by J. Pawluk and Son. NW¼, sec.8, Tp.53, R.23, W4.

Description	Average Thickness (ft)
Pleistocene	
Silt and clay, stratified	2.5
Till, brown, columnar jointing	12.5
Saskatchewan Gravels: sand	5+

APPENDIX C: DATA

FABRIC 1

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
353	10	006	00	350	15	060	80	335	03
015	00	005	00	353	00	345	10	270	15
020	00	075	45	040	00	223	03	340	22
230	00	013	30	030	25	240	05	237	11
065	25	020	22	070	00	173	10	205	15
320	06	255	10	165	00	340	20	355	25
355	00	340	05	030	10	000	20	325	15
205	30	140	20	145	15	335	10	010	00
060	25	150	35	355	00	100	20	175	35
120	50	100	35	080	00	355	25	065	25

FABRIC 2

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
020	00	060	00	220	25	155	25	205	10
045	65	010	00	050	30	025	30	060	00
110	05	030	35	275	45	220	25	310	15
170	45	045	25	330	05	015	10	005	25
030	35	220	25	045	20	030	20	045	55
325	30	045	25	050	15	265	15	035	10
090	10	030	25	010	10	215	40	310	40
345	05	025	25	025	25	035	20	330	15
080	40	250	05	340	25	035	10	010	05
160	30	065	55	020	20	335	20	325	15

FABRIC 3

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
275	25	010	25	135	75	220	65	020	10
345	15	035	15	040	15	075	20	175	10
065	10	305	15	260	20	185	10	200	40
100	30	135	20	335	05	070	15	115	75
350	20	155	15	105	30	055	00	020	25
050	35	025	05	065	70	260	05	035	20
010	10	095	20	120	10	010	15	075	35
120	40	085	20	345	05	070	15	090	45
065	15	045	30	060	30	260	25	080	45
000	15	265	30	355	15	180	10	090	30
270	30	135	10	160	05	175	20	355	40
010	25	150	05	255	45	190	15	120	35
310	10	060	15						

FABRIC 4

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
325	45	020	25	340	40	285	20	350	40
105	40	095	20	070	15	085	20	010	20
035	35	345	05	115	25	230	10	080	20
115	15	090	35	005	15	125	50	050	15
295	30	025	50	015	50	320	25	095	10
065	25	195	00	020	35	085	15	100	10
150	05	080	40	280	30	200	05	350	15
095	30	070	15	140	10	340	45	285	10
275	15	295	05	065	05	350	05	315	10
335	15	105	30	295	15	320	15	100	05

FABRIC 5

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
112	13	126	08	071	16	304	12	269	19
137	03	154	01	142	25	256	15	307	34
194	14	157	16	149	26	353	50	082	35
229	25	155	16	106	36	144	59	031	15
153	50	186	14	051	02	304	01	053	32
038	03	029	04	041	14	084	21	330	19
031	04	334	42	356	33	314	18	178	28
217	17	067	09	250	28	032	03	051	00
326	08	340	29	218	36	204	05	012	05
137	24	020	01	177	05	150	09	195	33
312	08	031	02						

FABRIC 6

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
086	17	252	02	238	02	204	05	002	05
089	18	121	24	037	06	090	23	252	03
125	67	213	29	300	06	157	67	253	10
062	44	080	10	054	07	062	04	329	68
221	14	247	03	229	05	153	15	122	10
078	13	222	01	263	06	285	42	238	25
225	22	064	05	023	08	234	02	319	36
093	10	349	09	321	08	050	40	257	45
050	43	104	09	106	14	265	17	256	12
022	12	078	21	296	32	129	01	099	40

FABRIC 7

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
349	07	305	02	011	31	031	03	029	17
006	12	019	00	086	12	341	08	350	06
056	24	312	08	047	01	063	12	044	03
355	22	052	03	014	26	048	06	034	08
031	12	099	00	064	11	070	24	175	18
190	11	052	02	315	06	250	03	111	02
307	10	038	02	033	05	081	00	044	00
191	11	195	09	052	01	050	11	019	02
020	03	021	03	079	27	017	01	126	02
095	00	260	33	003	01	041	07	053	09
059	02								

FABRIC 8

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
065	60	115	20	305	50	305	65	025	50
310	05	325	70	335	40	265	60	065	60
025	60	090	45	115	25	290	15	010	50
355	45	320	15	090	30	110	30	245	60
160	30	015	55	125	05	080	25	340	30
005	55	030	45	295	35	325	60	265	30
325	50	285	15	005	40	050	25	305	40
295	10	320	45	345	45	320	45	060	35
295	35	295	40	330	60	050	45	310	30
330	40	305	45	325	30	345	45	350	30

FABRIC 9

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
068	31	222	02	188	03	056	08	043	19
332	32	253	07	359	31	035	28	049	07
073	04	043	30	249	11	247	11	017	34
041	15	318	03	084	16	012	32	324	22
313	20	069	33	273	13	324	02	062	03
175	00	338	17	044	03	005	13	062	05
329	25	232	17	283	05	024	09	082	04
037	01	038	10	208	05	044	07	042	02
028	08	036	00	030	06	018	09	033	00
043	03	275	21	026	13	031	02	297	04

FABRIC 10

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
112	03	103	05	067	01	066	12	320	10
003	02	015	43	352	03	019	19	026	09
116	04	168	39	018	06	000	01	005	12
172	08	101	02	009	02	028	15	006	02
116	20	345	05	025	01	281	18	358	26
220	15	324	14	015	02	356	02	055	03
353	28	031	14	094	19	016	07	090	00
014	08	285	10	052	17	119	03	188	37
072	13	279	08	060	12	193	04	296	15
273	11	103	25	001	04	053	26	018	22

FABRIC 11

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
315	09	241	00	260	01	235	26	272	10
275	00	295	16	232	16	337	24	242	31
102	12	271	21	235	35	272	06	257	42
325	03	335	15	194	29	008	00	287	11
330	18	155	55	219	11	170	61	227	08
215	42	295	05	227	00	275	07	333	03
347	13	059	43	009	02	208	24	028	02
066	26	278	15	075	09	035	25	219	02
287	00	082	07	243	32	038	14	202	19
178	43	255	04	035	00	255	02	244	01

FABRIC 12

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
137	12	197	01	036	07	201	15	107	05
170	03	323	10	091	19	199	15	341	05
112	06	092	30	041	14	021	19	083	15
092	26	307	06	027	24	166	10	101	04
038	17	003	07	048	07	018	02	014	15
017	05	005	02	325	08	346	11	047	00
027	06	029	02	240	22	183	23	005	01
018	10	183	03	248	14	233	35	222	21
238	24	053	21	063	16	017	00	022	00
200	46	194	06	210	28	017	02		

FABRIC 13

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
085	11	140	20	320	06	125	31	335	09
280	23	240	23	005	00	165	00	165	03
345	14	060	31	045	14	110	22	290	18
195	03	100	08	145	13	215	30	350	01
015	06	315	11	335	25	215	08	130	13
090	35	335	13	130	06	150	20	115	16
090	19	315	16	295	15	315	20	000	27
095	21	205	08	090	34	135	13	315	19
150	17	135	13	120	01	190	08	150	08
155	01	125	24	210	04	035	16	225	09
015	04	050	20	145	24	025	15	110	22
135	20	305	11	125	19				

FABRIC 17

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
166	06	193	57	176	61	186	55	216	17
186	24	167	19	207	54	159	21	213	23
159	14	212	20	193	23	176	62	083	08
329	12	317	36	313	15	241	28	279	09
327	65	093	20	231	08	288	23	144	29
131	01	279	17	123	03	267	72	069	20
137	07	319	17	254	52	115	10	264	16
121	13	243	13	290	22	115	07	280	26
070	35	267	60	127	10	266	34	266	18
262	04	126	31	101	22	252	19	117	16

FABRIC 18

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
097	10	341	31	124	17	337	42	309	09
304	25	359	34	279	10	328	20	347	44
277	21	030	14	067	35	098	01	025	24
101	12	279	39	029	17	017	14	304	26
041	17	006	20	322	22	126	40	317	45
328	32	132	05	001	31	002	23	317	28
322	34	337	33	083	13	337	15	316	32
286	16	315	31	305	43	341	32	023	27
315	30	345	31	355	22	303	49	287	22
335	17	339	30	327	25	351	42	314	29

FABRIC 20

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
310	18	116	04	045	12	261	11	125	12
044	18	062	19	283	14	042	01	072	06
202	11	295	15	122	32	333	23	221	17
352	12	031	67	358	27	263	14	101	21
209	04	334	23	316	17	074	43	017	36
326	28	299	32	132	14	106	02	287	17
203	22	119	14	300	11	028	19	258	20
277	26	308	04	151	25	347	11	118	11
115	10	125	45	163	13	185	03	193	09
337	14	088	04	324	10	354	06	016	11

FABRIC 21

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
043	05	072	30	071	26	228	43	228	10
299	09	108	32	257	07	032	07	164	02
187	46	352	47	198	09	022	32	000	15
054	14	332	03	055	28	042	40	065	11
020	15	061	10	002	08	059	10	228	06
023	07	232	04	115	44	167	36	108	58
007	14	015	29	073	38	125	23	158	36
144	05	151	18	327	25	223	24	208	27
071	07	173	01	150	37	219	27	328	28
282	23	147	20	348	28	024	27	018	05

FABRIC 22

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
267	17	077	21	294	15	312	60	254	01
014	34	062	21	271	03	222	19	280	11
033	08	294	27	282	12	257	30	072	22
107	14	272	10	283	13	306	24	189	50
237	13	295	35	233	15	097	35	067	40
098	33	336	40	076	25	057	24	095	12
043	21	252	04	033	13	324	49	062	27
051	34	330	29	047	02	337	37	079	33
253	24	301	29	237	01	316	34	066	12
203	21	075	19	028	54	076	17	052	44

FABRIC 23

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
006	17	136	32	071	10	047	54	354	18
090	01	240	43	037	57	233	20	205	54
315	25	255	31	237	44	068	53	151	21
064	05	142	02	011	29	168	42	338	52
155	24	003	55	283	39	160	22	316	29
011	08	226	35	076	67	057	45	343	24
030	22	243	12	335	26	043	23	025	08
261	27	253	32	170	05	244	11	188	17
242	40	311	05	023	43	040	30	262	23
137	19	237	49	068	21	233	13	248	35

FABRIC 24

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
007	20	348	22	036	24	005	17	263	21
050	15	242	04	352	15	034	29	354	30
028	25	014	25	035	05	096	36	071	39
032	37	018	16	187	26	062	53	350	36
053	26	059	17	031	26	046	07	053	07
331	35	346	28	341	28	081	14	324	32
046	16	006	32	343	30	267	08	009	01
007	06	048	08	354	26	308	15	145	26
038	24	162	06	192	05	036	06	205	05
318	35	023	28	048	13	344	41	002	20

FABRIC 25

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
062	38	223	43	284	40	007	40	204	43
297	42	210	28	327	37	307	32	011	59
227	21	070	41	259	28	297	01	351	36
039	17	201	18	031	17	079	50	203	04
338	39	200	03	355	30	326	15	144	55
318	20	329	40	006	45	173	33	147	45
245	48	211	05	138	32	319	32	270	58
013	14	240	41	089	16	169	29	270	13
134	26	322	07	332	51	343	34	346	40
158	17	358	43	212	19	025	50	025	05

FABRIC 26

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
002	05	218	23	337	32	354	26	339	26
151	14	007	15	329	13	065	17	116	37
299	34	144	05	312	45	329	10	148	13
334	11	327	12	356	12	348	03	335	15
007	21	347	15	320	22	147	03	193	23
012	09	337	20	043	17	160	12	129	17
331	14	031	23	158	09	202	20	197	36
025	43	002	15	353	47	042	38	114	23
347	12	116	12	016	23	150	14	322	18
078	40	358	21	298	29	348	38	323	19

FABRIC 27

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
027	05	017	22	270	50	040	18	275	70
027	05	096	47	249	22	317	55	000	21
258	21	202	05	099	40	179	33	110	66
141	03	298	13	176	10	028	01	041	17
343	30	229	17	262	26	227	47	049	15
201	05	356	50	062	19	007	04	225	15
164	14	082	15	173	08	217	09	082	28
244	17	179	14	044	02	337	13	313	27
185	06	193	05	336	40	192	18	014	11
050	06	327	10	232	15	133	56	227	15

FABRIC 28

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
355	03	006	09	069	12	282	64	333	10
012	14	196	04	006	22	285	27	013	15
061	30	130	16	088	05	037	44	334	15
287	25	319	24	175	04	356	05	167	24
152	08	078	14	208	28	290	27	201	12
043	52	107	21	047	07	346	29	267	11
047	32	206	16	114	40	188	05	120	33
227	09	163	28	195	02	347	13	049	20
351	19	187	12	163	02	357	05	173	14
053	40	135	09	067	58	332	04	253	19

FABRIC 29

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
061	05	118	01	043	14	140	24	024	09
143	09	025	02	094	38	048	15	242	30
181	32	308	15	023	10	191	05	216	08
002	02	117	04	107	05	087	09	267	05
260	23	312	16	188	20	338	38	350	46
278	44	033	22	018	05	094	01	354	13
072	05	078	05	322	52	097	17	104	16
069	11	151	08	028	28	276	20	112	03
356	15	040	26	228	14	166	19	356	06
217	10	350	19	183	15	114	02	026	26

FABRIC 30

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
170	11	186	26	072	15	186	52	120	39
255	47	128	54	187	77	014	38	287	18
105	01	202	48	166	70	135	04	286	41
151	45	328	09	153	36	030	55	324	25
243	64	165	37	141	02	116	26	111	10
194	05	101	24	148	50	335	20	171	43
161	16	290	61	263	41	151	38	148	49
170	27	273	11	113	64	087	01	154	38
108	08	089	05	307	27	073	46	132	15
186	38	106	33	031	49	332	16	095	30

FABRIC 31

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
018	54	264	26	050	46	287	02	338	23
085	49	045	16	218	18	063	15	228	11
284	39	091	11	287	41	354	05	296	31
276	48	143	10	306	72	349	05	343	34
051	22	169	26	233	11	014	19	302	09
000	20	305	25	138	17	224	11	074	19
312	64	286	58	198	08	345	24	323	18
271	06	001	03	109	09	100	15	030	22
351	17	107	46	334	14	359	31	052	08
227	05	092	27	278	23	228	18	255	09

FABRIC 32

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
196	17	305	12	112	23	011	34	072	41
356	32	135	27	136	21	101	18	111	16
138	52	303	35	343	29	118	40	147	32
022	34	004	02	294	40	149	19	039	46
123	08	006	35	322	07	316	15	003	13
258	47	098	08	144	06	246	45	123	55
125	07	146	20	234	46	020	05	257	38
164	02	185	45	087	12	071	40	095	13
068	38	242	12	175	13	004	31	032	22
157	17	014	10	318	10	226	20	132	18

FABRIC 33

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
005	03	277	24	264	27	338	29	047	34
305	29	235	22	293	48	007	05	015	60
057	26	331	10	078	37	121	07	341	33
308	30	087	51	267	38	004	49	350	08
273	29	197	20	333	08	074	16	345	11
016	32	272	03	193	63	309	12	020	14
144	16	218	14	290	23	060	21	036	41
335	18	002	18	254	07	147	18	276	25
079	12	014	23	356	20	305	27	174	12
328	29	346	33	304	20	327	25	164	20

FABRIC 34 (Dimensions in millimeters)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
001	087	040	030	2.2	1.3	045	17		
002	028	016	011	1.8	1.5	117	13		
003	118	096	060	1.2	1.6	264	14		
004	052	037	029	1.4	1.3	065	07		
005	059	048	033	1.2	1.5	229	14		
006	059	041	035	1.4	1.2	064	33		
007	026	021	011	1.2	1.9	099	10		
008	030	018	018	1.7	1.0	196	29		
009	041	037	016	1.1	2.3	216	41		
010	040	029	022	1.4	1.3	091	09		
011	039	029	020	1.3	1.5	021	21		
012	024	015	012	1.6	1.3	104	10		
013	026	014	012	1.9	1.2	024	74		
014	048	033	025	1.5	1.3	048	26		
015	068	055	045	1.2	1.2	354	17		
016	066	062	037	1.1	1.7	335	08	170	34 NE
017	060	038	028	1.6	1.4	319	01	152	32 NE
018	035	026	016	1.3	1.6	008	04	117	07 NE
019	031	026	014	1.2	1.9	054	14	028	38 SE
020	017	010	007	1.7	1.4	199	27		
021	019	014	011	1.4	1.3	217	03		
022	066	053	047	1.2	1.1	185	18		
023	035	025	021	1.4	1.2	061	34		
024	027	020	016	1.4	1.3	309	09		
025	014	011	008	1.3	1.4	280	56		
026	022	016	014	1.4	1.1	025	18		
027	031	027	018	1.1	1.5	260	05		
028	036	025	013	1.4	1.9	359	29	168	68 NE
029	045	025	020	1.8	1.3	027	10		
030	053	030	013	1.8	2.3	114	48		
031	023	015	013	1.5	1.2	053	36		
032	021	014	010	1.5	1.4	209	36		
033	075	065	041	1.2	1.6	208	13		
034	033	032	024	1.0	1.3	065	08		
035	027	018	011	1.5	1.6	017	02		
036	025	021	015	1.2	1.4	297	43		
037	020	012	010	1.7	1.2	233	79		
038	024	020	009	1.2	2.2	137	31	047	31 SE
039	055	048	034	1.1	1.4	220	12		
040	030	020	017	1.5	1.2	249	11		

FABRIC 34 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
041	030	026	018	1.2	1.4	058	41	058	90
042	028	024	017	1.2	1.4	123	75		
043	023	013	011	1.8	1.2	035	12		
044	038	028	019	1.4	1.5	176	11		
045	083	061	050	1.4	1.2	206	15		
046	036	018	016	2.0	1.1	230	46		
047	047	034	026	1.4	1.3	027	05		
048	025	023	012	1.1	1.9	054	64		
049	027	021	019	1.3	1.1	030	06		
050	026	021	009	1.2	2.3	228	38	092	49 SW
051	017	012	010	1.4	1.2	108	34		
052	023	015	010	1.5	1.5	066	80		
053	032	024	015	1.3	1.6	136	23		
054	033	022	014	1.5	1.6	145	53		
055	028	028	021	1.0	1.3			060	68 SE
056	032	018	018	1.8	1.0	246	06		
057	032	028	016	1.1	1.7	357	20	031	36 NW
058	041	032	016	1.3	2.0	094	05	054	09 SE
059	042	036	029	1.2	1.2	238	17		
060	029	024	020	1.2	1.2	299	30		
061	031	028	015	1.1	1.9	115	40	135	63 NE
062	038	030	017	1.3	1.8	112	15	085	19 SE
063	030	017	005	1.8	3.4	269	12	076	46 NW
064	143	115	067	1.2	1.7	046	21		
065	038	030	021	1.3	1.4	051	15		
066	054	041	015	1.3	2.7	238	02	044	13 NW
067	034	022	015	1.5	1.5	200	13		
068	031	020	013	1.6	1.5	202	30		
069	028	017	015	1.6	1.1	279	02		
070	026	025	015	1.0	1.7			017	50 SE
071	028	019	009	1.5	2.1	306	04	036	04 NW
072	021	016	008	1.3	2.0	218	21	055	59 SE
073	031	024	019	1.3	1.3	027	08		
074	036	027	017	1.3	1.6	028	23		
075	033	024	018	1.4	1.3	038	18		
076	030	019	013	1.6	1.5	008	04		
077	086	068	058	1.3	1.2	169	24		
078	025	013	009	1.9	1.4	057	32		
079	038	021	018	1.8	1.2	083	34		
080	029	020	008	1.5	2.5	077	06	102	22 NE

FABRIC 34 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
081	040	029	012	1.4	2.4	042	08	132	08 NE
082	058	033	028	1.8	1.2	013	10		
083	039	026	017	1.5	1.5	341	35	135	40 NE
084	043	040	021	1.1	1.9	027	04	045	07 NW
085	028	018	015	1.6	1.2	027	029		
086	035	027	023	1.3	1.2	073	26		
087	019	017	013	1.1	1.3	261	11		
088	024	017	015	1.4	1.1	064	35		
089	025	020	015	1.3	1.3	284	18		
090	033	028	020	1.2	1.4	336	56		
091	028	016	014	1.7	1.1	318	09		
092	072	049	032	1.5	1.5	182	38		
093	043	034	031	1.3	1.1	148	50		
094	042	033	033	1.3	1.0	007	10		
095	018	011	008	1.6	1.4	087	19	177	19 NE
096	024	017	006	1.4	2.8	027	09		
097	033	023	021	1.4	1.1	193	10		
098	034	022	010	1.5	2.2	021	13	078	15 NW
099	039	036	018	1.1	2.0			174	21 SE
100	032	030	018	1.1	1.7	022	04	040	13 NW
101	047	032	012	1.5	2.7	022	15	022	85 NW
102	030	020	015	1.5	1.3	017	20		
103	032	026	021	1.2	1.2	237	19		
104	078	053	034	1.5	1.6	202	05	137	07 SW
105	034	033	013	1.0	2.5			025	78 NW
106	016	012	006	1.3	2.0	218	13	033	66 NW
107	016	012	008	1.3	1.5	072	15	086	69 NW
108	018	015	012	1.2	1.3	349	22		
109	061	041	027	1.5	1.5	345	36	114	49 NE
110	032	021	008	1.5	2.6	337	21	106	30 NE
111	026	020	016	1.3	1.3	099	07		
112	020	013	009	1.5	1.4	261	10		
113	035	030	021	1.2	1.4			117	85 NE
114	024	021	010	1.1	2.1	065	07	040	24 SE
115	033	016	014	2.1	1.1	282	11		
116	021	017	013	1.2	1.3	289	46		
117	029	024	017	1.2	1.4	233	20		
118	029	019	018	1.5	1.1	242	18		
119	053	042	024	1.3	1.8			164	73 SW
120	074	070	051	1.1	1.4	270	23		

FABRIC 34 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
121	031	024	012	1.3	2.0	028	22	028	90
122	017	016	006	1.1	2.7			016	13 SE
123	032	022	018	1.5	1.2	177	08		

FABRIC 35 (Dimensions in millimeters)

001	057	047	026	1.2	1.8	251	04	038	34 NW
002	030	025	019	1.2	1.3	354	16		
003	024	019	014	1.3	1.4	236	63	236	90
004	017	013	007	1.3	1.9	348	49	023	60 NW
005	019	013	007	1.5	1.9	218	44	087	50 SE
006	028	023	017	1.2	1.4	318	75	209	77 NW
007	037	030	024	1.2	1.3	165	28		
008	015	011	006	1.4	1.8	295	76		
009	016	014	008	1.1	1.8	030	49		
010	020	016	012	1.3	1.3	002	06	185	70 NW
011	018	015	008	1.2	1.9	111	17	117	17 NE
012	020	010	009	2.0	1.1	224	34		
013	031	024	016	1.3	1.5	200	03		
014	025	015	012	1.7	1.3	349	25	149	37 NE
015	035	025	018	1.4	1.4	316	14	316	90
016	056	045	024	1.2	1.9	196	21	199	84 NW
017	025	016	012	1.6	1.3	192	15	032	61 SE
018	019	014	013	1.4	1.1	342	37		
019	023	012	011	1.9	1.1	215	16		
020	028	019	011	1.5	1.7	054	05	098	19 NE
021	020	014	009	1.4	1.6	358	23		
022	022	013	007	1.7	1.9	222	29	224	69 NW
023	032	024	015	1.3	1.6	243	21	252	69 NW
024	026	015	011	1.7	1.4	039	14	255	48 NW
025	031	028	016	1.1	1.8	075	10	108	22 NE
026	016	009	008	1.8	1.1	186	11		
027	019	010	008	1.9	1.3	215	14		
028	053	043	037	1.2	1.2	150	30		
029	033	024	008	1.4	3.0			199	77 SE
030	040	031	020	1.3	1.6	201	07		

FABRIC 35 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
031	030	022	011	1.4	2.0	113	12	078	14 SE
032	024	017	009	1.4	1.9	007	09	008	52 SE
033	040	030	025	1.3	1.2				
034	018	013	009	1.4	1.4	051	16		
035	017	012	008	1.4	1.5	017	26		
036	030	024	014	1.3	1.7	252	49	178	62 SW
037	045	037	030	1.2	1.2	027	34		
038	030	022	014	1.4	1.6	033	21	213	85 NW
039	025	018	010	1.4	1.8	029	25	259	34 NW
040	035	030	024	1.2	1.3				
041	030	025	021	1.2	1.2	234	13		
042	015	013	007	1.2	1.9	154	10	092	15 SW
043	031	029	025	1.1	1.2				
044	023	019	011	1.2	1.7	046	31	025	58 SE
045	019	012	009	1.6	1.3	220	11		
046	029	019	017	1.5	1.1	102	24		
047	017	014	009	1.2	1.6	157	40		
048	039	025	017	1.6	1.5	048	12		
049	029	023	020	1.3	1.2	241	41		
050	015	011	008	1.4	1.4	351	42		
051	038	032	021	1.2	1.5	001	29		
052	020	017	009	1.2	1.9	041	48	024	82 SE
053	023	018	015	1.3	1.2	211	18		
054	025	018	014	1.4	1.3	269	64		
055	034	032	021	1.1	1.5			032	51 SE
056	032	024	021	1.3	1.1	342	06		
057	069	036	031	1.9	1.2	319	35		
058	069	043	041	1.6	1.0	199	30		
059	015	010	005	1.5	2.0	332	51		
060	025	018	008	1.4	2.3	332	16		
061	018	013	008	1.4	1.6	313	22		
062	015	010	008	1.5	1.3	070	08		
063	013	010	005	1.3	2.0	008	42		
064	010	008	005	1.3	1.6	224	03		
065	031	023	018	1.3	1.3	254	57		
066	020	013	008	1.5	1.6	195	11		
067	020	018	005	1.1	3.6	285	33		
068	018	013	013	1.4	1.0	336	37		
069	059	046	038	1.3	1.2	233	29		
070	025	020	010	1.3	2.0	006	17		

FABRIC 35 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
071	030	023	013	1.3	1.8	159	01		
072	018	013	010	1.4	1.3	025	55		
073	020	018	010	1.1	1.8	324	21		
074	028	018	008	1.6	2.3	346	13		
075	025	020	015	1.3	1.3	238	76		
076	030	020	018	1.5	1.1	216	05		
077	018	013	010	1.4	1.3	010	37		
078	064	046	025	1.4	1.8	074	37		
079	043	030	030	1.4	1.0	214	33		
080	020	015	008	1.3	1.9	221	32		
081	033	015	015	2.2	1.0	033	52		
082	043	030	020	1.4	1.5	028	21		
083	020	013	010	1.5	1.3	012	18		
084	030	025	023	1.2	1.1	008	37		
085	025	020	013	1.3	1.5	217	03		
086						239	24		
087						250	05		
088						358	30		
089						196	36		
090						019	12		
091						024	15		
092						185	09		
093						200	06		
094						173	05		
095						010	21		
096						227	28		
097						033	44		
098						277	13		
099						235	18		
100						199	22		
101						225	17		
102						258	37		
103						217	02		
104						027	28		
105						256	13		
106						285	54		

FABRIC 36 (Dimensions in millimeters)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
001	039	031	023	1.3	1.3	175	01	129	18 NE
002	065	058	047	1.1	1.2	072	20		
003	020	015	013	1.3	1.2	022	27		
004	029	023	018	1 3	1.3	014	33		
005	117	075	061	1.6	1.2	016	08		
006	069	061	036	1.1	1.7	011	23	137	40 NE
007	150	106	085	1.4	1.2	214	01		
008	029	018	012	1.6	1.5	114	02	110	50 NE
009	025	020	012	1.3	1.7	011	16	203	82 NW
010	021	018	015	1.2	1.2	348	43		
011	027	025	018	1.1	1.4	117	38		
012	037	025	019	1.5	1.3	087	41		
013	043	030	025	1.4	1.2	049	41		
014	035	029	024	1.2	1.2	136	68		
015	040	024	018	1.7	1.3	344	14		
016	033	024	023	1.4	1.0	012	65		
017	025	017	013	1.5	1.3	048	36		
018	033	021	006	1.6	3.5	083	14	107	24 NE
019	022	017	012	1.3	1.4	341	45		
020	064	030	024	2.1	1.3	358	11		
021	235	137	096	1.7	1.4	210	01		
022	043	034	016	1.3	2.1	235	13	192	21 NW
023	023	016	012	1.4	1.3	107	20		
024	030	027	016	1.1	1.7			115	40 NE
025	038	021	020	1.8	1.1	343	26		
026	031	025	016	1.2	1.6	109	10	115	39 NE
027	022	018	009	1.2	2.0	057	28	127	35 NE
028	034	024	020	1.4	1.2	075	31		
029	066	030	020	2.2	1.5	023	24	217	59 NW
030	016	013	004	1.2	3.3	053	39	267	43 NW
031	020	015	011	1.3	1.4	333	27		
032	020	020	008	1.0	2.5			006	69 SE
033	020	012	008	1.7	1.5	349	08		
034	038	025	022	1.5	1.1	343	27		
035	048	037	013	1.3	2.8	213	14	203	53 NW
036	048	029	022	1.7	1.3	321	18		
037	052	040	027	1.3	1.5	356	26		
038	017	011	011	1.5	1.0	205	03		
039	033	018	014	1.8	1.3	349	08		
040	028	011	005	2.5	2.2	009	27	117	29 NE

FABRIC 36 (continued)

Pebble number	Axial lengths a	b	c	Axial ratios a:b	b:c	a axis trend	pl	ab plane str	dip
041	035	023	018	1.5	1.3	047	25		
042	046	025	024	1.8	1.0	339	28		
043	023	018	008	1.3	2.3	018	09	092	13 NE
044	018	013	007	1.4	1.9	049	30	139	30 NE
045	055	040	031	1.4	1.3	037	27		
046	028	021	010	1.3	2.1	044	11	090	21 N
047	038	024	015	1.6	1.6	070	05	070	90
048	043	025	017	1.7	1.5	356	30		
049	054	036	016	1.5	2.3	346	14	134	28 NE
050	029	021	009	1.4	2.3	107	05	107	73 NE
051	036	027	024	1.3	1.1	016	18		
052	036	028	020	1.3	1.4	019	23		

FABRIC 37 (Read across rows for original sequence)

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
014	22	056	12	206	10	202	06	007	64
208	34	083	13	292	19	092	12	017	45
356	15	298	12	333	28	201	22	258	02
223	10	287	24	051	25	068	13	307	20
193	03	068	10	064	25	204	15	010	14
216	18	035	21	205	12	292	50	324	05
111	08	266	12	215	21	024	37	141	21
251	27	065	27	343	28	059	33	283	17
201	14	320	04	184	30	063	24	236	28
044	22	294	27	138	09	338	24	044	40
344	22	276	03	083	18	030	38	153	41
080	11	210	05	070	28	016	15	304	28
237	39	141	19	326	57	004	43	304	47
195	14	335	21	200	01	306	07	245	05
317	08	330	24	311	21	335	04	333	26
111	04	304	09	291	35	270	15	157	40
160	51	281	23	066	12	221	19	209	53
090	09	116	54	308	21	191	21	307	13
123	09	152	04	316	01	296	02	352	25
123	07	297	20	049	19	282	01	119	22

FABRIC 38

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
054	19	352	12	151	08	102	35	047	33
112	06	043	43	132	08	057	37	075	10
068	17	087	21	319	20	328	24	019	05
313	32	091	33	103	23	047	22	339	25
107	05	334	08	078	35	024	15	049	20
323	10	052	31	359	03	004	30	066	50
153	05	337	07	276	15	058	08	292	09
060	15	089	14	204	05	045	27	097	36
084	29	071	36	354	25	013	24	031	14
333	26	028	45	317	32	242	27	003	27

FABRIC 39

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
342	15	038	21	347	05	100	21	156	16
309	09	138	03	129	02	061	09	135	15
334	06	218	26	324	04	208	08	050	10
167	06	123	17	061	29	067	32	014	01
317	07	003	04	035	31	025	51	328	09
168	13	095	11	030	66	060	12	008	05
262	05	141	23	175	27	165	07	083	23
052	18	214	04	212	40	261	23	208	08
318	07	040	10	348	18	224	23	053	01
011	26	202	08	102	19	218	06	057	30

FABRIC 40

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
201	07	301	26	001	31	242	11	288	01
052	13	224	17	044	08	107	10	299	19
200	07	005	29	255	13	041	12	331	09
068	43	097	12	279	18	034	48	199	08
013	17	144	03	062	17	075	17	041	22
100	20	148	08	030	16	039	30	061	41
014	17	045	13	036	10	225	08	055	10
075	18	342	08	050	09	045	20	056	12
284	05	210	01	088	15	302	09	018	28
048	13	039	14	058	09	155	16	029	23

FABRIC 41

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
227	33	029	13	257	01	048	27	226	43
205	44	354	13	207	05	250	16	201	08
080	15	316	07	043	27	079	13	038	15
089	08	031	22	271	04	033	11	110	01
220	20	027	35	198	08	113	41	199	22
358	13	209	20	333	25	291	29	077	10
004	22	031	19	273	14	023	15	001	20
221	40	005	03	195	18	233	50	322	03
232	40	307	10	227	23	103	09	043	10
112	18	128	15	149	38	300	39	345	04

FABRIC 42

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
004	13	307	55	283	09	355	20	332	37
037	03	003	29	354	21	230	61	308	26
075	25	015	25	317	20	108	14	161	12
019	56	310	10	045	31	068	18	100	05
301	13	195	55	321	22	076	01	093	15
330	55	041	36	355	33	266	16	265	05
040	17	089	43	323	17	176	04	028	02
321	20	034	06	324	37	077	49	105	07
238	10	084	10	028	43	356	20	016	04
345	07	307	04	096	09	357	42	348	37

FABRIC 43

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
289	02	280	33	345	24	349	10	339	40
153	09	218	14	339	53	346	33	183	03
138	03	010	59	309	11	338	32	346	32
220	00	196	02	012	16	048	46	312	46
174	10	032	18	145	12	337	55	014	41
008	24	318	45	193	13	081	16	033	05
252	63	068	18	314	04	083	15	030	13
041	14	012	05	007	17	148	10	304	42
000	39	173	15	302	09	110	42	001	40
302	45	334	13	020	08	048	22	002	09

FABRIC 44

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
015	15	313	24	150	05	185	05	330	18
348	16	351	33	334	31	346	14	324	08
127	12	068	51	101	27	315	05	350	27
027	29	146	02	191	02	054	06	085	04
233	11	034	02	347	03	000	27	045	30
300	33	320	30	319	18	346	01	050	05
021	05	277	06	010	21	318	06	015	16
353	02	351	04	282	37	357	25	345	13
280	06	075	30	332	17	000	04	033	07
350	23	018	01	347	05	352	07	127	06

FABRIC 45

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
160	10	190	15	080	15	250	25	325	15
250	05	300	20	285	05	030	25	300	05
105	05	320	15	110	00	290	20	010	30
310	15	215	15	345	30	210	25	275	25
065	10	220	05	000	05	285	20	125	10
135	10	000	20	240	05	020	10	225	25
230	05	000	10	140	10	300	15	160	10
145	10	240	20	345	20	280	45	190	20
330	20	125	25	240	10	300	15	275	15
330	40	195	05	190	05	275	05	185	10
060	20	305	05	090	05	165	15	055	20
240	05	265	05	280	05	095	30	035	20
335	15	115	15	340	30	275	15	310	20
270	15	145	15	130	35	305	15	090	50
125	30	040	20	100	15	325	05	350	25

FABRIC 466-1

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
342	23	025	11	333	15	332	24	008	28
340	06	348	24	335	31	336	14	343	14
135	12	059	00	053	18	354	22	008	04
265	15	023	15	011	21	058	14	330	17
005	13	030	24	219	03	028	07	244	00
065	08	040	33	075	27	043	01	056	07
195	12	359	08	022	03	037	09	024	19
336	13	028	02	197	06	344	00	041	13
029	06	350	11	323	02	017	16	357	08
339	19	043	07	037	09	198	06	041	14

FABRIC 466-2

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
199	10	359	15	200	10	075	37	035	35
170	21	242	10	339	35	204	42	195	09
251	05	027	08	192	02	070	14	080	17
344	15	220	19	038	30	025	07	027	13
025	18	216	19	045	17	037	16	025	17
037	01	027	10	040	03	195	05	095	32
360	03	221	36	018	16	045	11	210	04
020	10	017	18	225	30	075	21	173	27
243	34	097	16	007	11	190	02	174	14
040	36	027	32	039	15	095	28	020	12

FABRIC 466-3

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
217	09	217	20	348	09	297	03	356	03
150	30	035	24	005	25	215	19	110	23
257	46	050	15	338	40	015	32	190	35
005	14	180	20	165	47	110	36	022	00
180	22	002	22	260	33	090	05	152	37
165	79	174	31	215	04	035	02	315	00
212	07	155	39	345	00	160	15	200	22
009	39	222	05	182	13	211	14	115	09
185	32	132	40	032	06	062	13	179	36
210	31	125	42	273	30	160	25	335	16

FABRIC 466-4

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
041	00	026	00	040	00	056	31	055	00
042	00	063	32	056	00	075	20	067	00
051	00	066	27	040	00	057	00	062	47
232	37	049	00	053	28	027	00	076	41
029	45	042	00	105	00	065	24	040	33
200	15	022	00	025	24	060	00	219	10
100	13	025	00	206	10	013	18	038	00
023	20	031	20	060	00	034	00	047	00
008	26	022	08	024	00	352	00	340	00
042	00	044	23	025	00	038	00	036	00

FABRIC 466-5

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
020	38	066	14	273	78	013	17	078	20
246	09	104	30	036	36	216	21	212	23
060	11	124	00	042	18	200	42	003	55
118	24	208	15	142	04	130	26	101	26
095	42	145	12	250	37	160	20	140	02
169	14	103	46	234	24	149	12	285	05
110	23	205	03	022	50	140	70	226	24
085	30	123	02	128	08	074	18	238	27
099	24	122	09	208	02	022	29	296	16
102	06	066	08	165	01	139	10	092	26
162	33								

FABRIC 466-6

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
140	11	340	55	162	12	330	34	350	27
358	36	155	39	148	05	315	04	222	10
200	07	030	08	097	42	092	02	110	24
035	10	128	22	220	23	040	02	112	41
117	23	035	10	245	40	253	23	275	25
093	12	283	18	195	10	262	12	234	27
277	43	270	15	215	05	110	10	274	15
175	10	010	04	063	05	012	01	055	03
073	14	055	23	155	14	232	28	080	31
220	25	210	13	310	05	155	14	100	05

FABRIC 466-7

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
065	15	270	10	080	22	235	08	260	05
129	10	115	29	256	52	087	16	048	30
095	35	131	38	042	54	052	61	332	50
062	27	014	29	093	16	108	21	092	43
089	48	047	35	025	54	304	22	110	31
014	20	098	42	097	63	067	06	104	48
175	39	305	30	053	18	045	35	228	15
222	36	018	46	133	07	334	43	205	42
034	27	071	05	285	56	074	36	045	34
348	53	217	32	012	27	048	12	126	04

FABRIC 466-8

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
153	15	150	12	160	05	185	00	015	14
000	05	225	05	030	15	115	25	190	15
315	10	190	05	233	28	176	10	205	10
150	10	175	16	225	07	215	20	215	05
205	20	130	20	150	29	200	05	210	17
320	06	240	18	165	00	195	00	035	13
195	30	210	10	200	35	215	23	205	24
190	21	195	55	215	10	230	05	230	12
215	07	120	00	190	00	215	10	205	00
205	10	185	10	030	15	155	44	140	35

FABRIC 466-9

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
215	20	265	04	130	25	062	07	300	10
320	21	250	01	240	18	110	19	115	01
246	07	190	12	010	22	310	30	355	29
025	10	190	09	220	38	283	13	315	18
110	40	055	10	084	01	140	08	260	14
040	16	315	25	135	34	065	24	083	41
230	09	165	18	155	01	290	24	140	15
160	03	127	12	123	08	296	38	051	10
180	16	265	08	200	01	320	20	020	23
275	50	304	30	148	05	302	14	115	29

FABRIC 466-10

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
015	20	020	41	295	30	017	00	007	23
035	22	348	07	065	35	005	51	359	03
043	11	085	05	310	10	340	33	010	00
350	11	305	16	325	06	300	08	300	10
290	09	288	12	330	07	283	03	350	00
040	16	350	20	070	10	285	10	055	15
180	60	235	17	160	25	252	22	250	35
200	27	280	14	325	15	334	08	265	02
300	05	326	03	330	22	308	02	295	02
005	05	325	21	346	05	098	39	290	10

FABRIC 466-11

Az	PI	Az	PI	Az	PI	Az	PI	Az	PI
066	50	045	50	002	04	055	37	046	24
068	44	086	40	071	16	102	37	130	20
042	27	018	56	028	33	034	02	074	38
092	46	005	25	048	05	052	04	055	08
003	02	085	35	072	32	012	11	034	08
018	28	006	08	005	30	010	09	010	10
290	15	010	39	275	26	015	42	345	30
155	90	200	35	240	42	200	12	200	05
145	16	190	08	160	43	275	30	280	34
220	57	170	17	145	45	145	04	245	31

FABRIC 466-12

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
285	08	275	02	000	03	010	36	350	12
350	43	010	17	230	16	010	22	010	03
216	21	008	05	280	22	110	26	012	52
030	46	030	42	000	39	030	24	020	14
010	02	350	02	270	20	280	24	330	17
012	11	080	02	000	48	025	08	009	11
310	40	000	15	000	11	340	08	300	02
015	04	320	16	040	04	350	07	025	07
290	08	000	21	300	07	340	31	000	31
010	22	010	26	030	14	310	25	310	27

SAMPLE 505

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
359	23	352	67	347	70	346	44	342	35
340	71	339	22	327	13	315	67	314	31
314	83	299	56	293	56	289	25	280	22
274	23	263	06	259	24	258	06	225	32
224	69	208	40	188	20	187	18	182	08
177	27	176	19	175	17	172	30	159	26
153	53	142	79	126	09	107	02	101	09
088	45	083	62	083	03	073	21	068	69
067	47	049	05	042	12	041	36	040	20
029	29	016	01	009	11	002	02	001	25

SAMPLE 506

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
348	15	338	38	319	35	313	01	310	09
294	10	291	64	291	55	283	08	275	57
261	55	260	58	258	73	256	11	253	47
245	50	241	48	219	10	215	22	206	06
202	48	202	09	192	19	190	73	184	53
174	55	165	28	146	56	140	67	128	71
127	43	116	60	116	04	113	65	109	71
080	60	073	01	069	17	063	37	057	40
047	63	044	06	040	05	036	65	025	19
023	61	011	42	009	12	006	04	005	38

SAMPLE 507

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
000	40	001	30	030	08	033	10	041	09
048	68	055	47	060	27	063	21	065	38
075	45	082	65	085	29	088	02	090	65
091	15	097	14	103	02	105	30	109	16
110	12	120	54	122	74	148	40	154	39
157	01	158	45	180	23	194	76	198	74
198	23	202	42	207	58	210	10	222	37
241	44	242	24	247	56	261	09	266	45
270	11	278	03	281	19	292	35	315	45
320	51	344	30	350	12	351	22	356	43

SAMPLE 508

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
001	25	007	20	011	27	017	15	018	25
023	01	029	30	035	08	047	10	049	02
051	24	053	05	064	07	065	17	079	28
089	17	090	29	120	01	122	16	125	05
130	17	136	06	145	05	145	21	153	02
156	27	163	01	177	26	178	23	189	03
199	21	214	19	218	09	239	02	241	21
243	08	245	17	261	24	264	06	279	03
283	21	289	12	290	18	299	30	322	20
335	01	351	03	353	19	356	04	356	09

SAMPLE 509

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
005	19	007	08	015	16	020	09	020	15
024	26	026	19	030	08	034	03	034	08
035	02	045	27	062	28	068	25	078	04
081	01	087	07	092	13	094	19	098	20
102	11	135	13	136	26	139	28	142	25
147	08	152	03	161	14	171	12	181	13
188	28	194	10	202	21	218	06	221	07
222	09	236	19	243	22	244	14	248	15
269	11	272	21	274	10	286	20	297	11
302	18	319	16	328	29	334	20	354	12

SAMPLE 510

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
001	25	004	23	011	16	013	23	025	17
039	02	041	13	051	05	068	15	078	04
081	08	098	24	111	06	117	07	124	14
131	09	135	10	137	13	139	29	142	20
163	26	173	12	181	08	187	20	194	20
206	25	210	08	210	22	212	08	220	08
223	27	228	13	236	17	244	03	247	12
257	21	270	29	271	13	277	04	281	18
282	23	283	06	296	19	298	08	307	08
324	12	330	04	337	28	350	18	357	29

SAMPLE 511

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
003	04	008	20	011	13	011	22	026	23
048	01	053	15	054	20	059	03	062	02
064	02	076	12	076	19	097	30	104	10
105	30	119	12	135	06	137	24	143	12
156	28	157	20	159	28	163	19	171	01
185	06	187	05	202	15	204	10	224	06
227	01	237	03	238	18	243	01	247	04
250	09	262	14	268	27	269	06	277	22
281	05	292	27	301	22	302	11	304	19
311	06	312	09	315	22	350	01	353	04

SAMPLE 512

Az	Pl	Az	Pl	Az	Pl	Az	Pl	Az	Pl
001	20	001	29	003	25	009	12	021	15
024	28	030	25	047	13	049	10	050	17
054	12	055	18	057	24	059	24	062	15
079	18	080	30	091	17	096	27	125	13
127	30	155	20	158	25	169	15	171	12
176	06	207	22	208	17	208	21	232	09
233	15	263	22	269	13	273	13	294	07
299	19	307	00	309	16	310	07	321	19
322	17	325	06	331	14	335	01	347	02
348	19	349	11	353	23	357	15	357	15

APPENDIX D:

COMPUTER PROGRAMS

Unmodified Point Density Program

This program plots the density of axes or the poles to planes on an equal-area projection. The area of the counting circle is specified as P, a percentage of the area of the projection. If α is the angular radius of the counting circle, it can be shown that

$$\cos \alpha = 1 - \frac{P}{100} .$$

The cosine of the angle ϕ between a grid point with direction cosines (l_1, m_1, n_1), and an observation with direction cosines (l,m,n) is given by

$$\cos \phi = l.l_1 + m.m_1 + n.n_1 .$$

Direction cosines (l,m,n) of an axis specified by its trend (TR) and plunge (PL) are given by

$$l = \cos(TR)\cos(PL)$$

$$m = \sin(TR)\cos(PL)$$

$$n = \sin(PL) .$$

During the counting procedure at each grid point, $\cos \phi$ is calculated for each observation and compared with $\cos \alpha$. If $\cos \phi \geq \cos \alpha$ (that is, $\phi \leq \alpha$) the point counter is incremented. The density values obtained are plotted as a grid of 333 numbers which can then be contoured by hand.

Modified Point Density Program

This modified procedure was described in some detail in Chapter 2. The approximation of the probability density surface for a true axis employs an expression derived by Watson and Irving (1957, p.293). They showed that, for large K, the probability Pr between R_1 and R_2 from the centre of the distribution is given by

$$Pr = e^{-K(1-\cos R_1)} - e^{-K(1-\cos R_2)} .$$

Using the specified value of K, Pr is calculated for successive radial increments of one degree. The average probability density (that is, probability per unit area) within each radial increment is then calculated. The areas are determined from the expression

$$A = 2 a^2(1-\cos R)$$

where A is the area of a circle of radius R on the surface of a sphere of radius a. Calculations involving spherical probability distributions are normally referred to a sphere of unit radius, so that a=1.

Successive values of probability density are subtracted to obtain the thicknesses of the disks used to approximate the probability distribution (see Chapter 2).

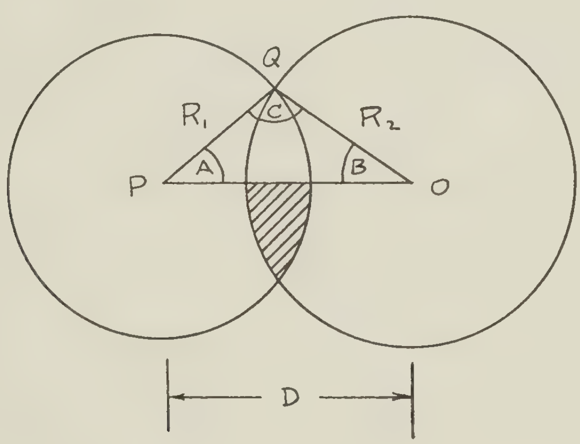
Using the radius of the counting circle calculated for the specified percent area, P, the volume of the probability distribution lying within the counting circle is calculated for centre-to-centre distances of zero, one degree, two degrees, and so on until the largest disk no longer overlaps the counting circle. These values of volume (that is, probability) are stored for use during the counting procedure.

The counting procedure is similar to that used by the unmodified

program, except that the angle ϕ, instead of being compared with the radius of the counting circle, is rounded off to the nearest degree and referred to the table of probabilities (volumes), and the appropriate probability used to increment the counter.

Common Area of Intersecting Circles on a Sphere

The calculation of volume referred to above involves, for a given centre-to-centre distance, computation of the common area of the counting circle and each disk of the approximated distribution. An expression for this area was derived as follows. Let the diagram below represent two circles of radii R_1 and R_2 with centre-to-centre distance D on the surface of a sphere of unit radius.



The angles A, B, and C are given by identities of spherical trigonometry:

$$\cos A = \frac{\cos R_2 - \cos R_1 \cos D}{\sin R_1 \sin D}$$

$$\cos B = \frac{\cos R_1 - \cos R_2 \cos D}{\sin R_2 \sin D}$$

$$\cos C = \frac{\cos D - \cos R_1 \cos R_2}{\sin R_1 \sin R_2}$$

The area of the triangle OPQ is then $A+B+C-\pi$ where A, B and C are in

radians. The area of circle 1 is $2\pi(1-\cos R_1)$ and that of the segment with angle A is $A(1-\cos R_1)$. Similarly, the area of the segment of circle 2 defined by the angle B is $B(1-\cos R_2)$. The sum of the areas of these two segments exceeds the area of the triangle OPQ by the area of the shaded portion in the diagram, which is half the common area of the circles. Thus

$$\begin{aligned}\tfrac{1}{2}(\text{common area}) &= (\text{area of segment A}) + (\text{area of segment B}) \\ &\quad - (\text{area of triangle OPQ}) \\ &= A(1-\cos R_1) + B(1-\cos R_2) - (A+B+C-\pi) \\ &= \pi - C - A\cos R_1 - B\cos R_2\end{aligned}$$

Hence

$$\text{common area} = 2(\pi - C - A\cos R_1 - B\cos R_2) .$$

Mean Vector Program

This program takes a set of N axes or poles to planes and, using a specified reference line and angular distance, selects those lying within the specified angle of the reference line. Using these n observations, it calculates the direction and length (R) of the resultant, and estimate (k) of the precision parameter of the population, the 95% and 99% confidence radii on the direction of the mean, and the vector magnitude (R/n), using the formulas given by Watson and Irving (1957).

After the mean direction has been calculated, all the data are in effect rotated until the mean is vertical. For each observation the computer then calculates (1) the complement of the new plunge and (2) the new trend referred to the original trend of the mean as zero azimuth. The trends are then grouped into 72 five-degree classes, and the plunge complements into 12 concentric or polar classes each equal to one twelfth of the radius of the limiting cone. Using the calculated precision parameter k, the computer then derives the expected frequencies in the polar classes assuming a Fisher distribution. Output from this part of the program consists of the polar class limits, expected frequencies in polar classes, observed frequencies in polar classes, and the observed frequencies in azimuthal classes.

The next stage in the program calculates chi-square values for the azimuth distribution, employing a procedure designed to overcome the problem posed by elongate and double groupings. Such groupings, while having non-random azimuthal distributions about their central direction, will give nearly equal frequencies in four quadrants about

a central point when the quadrants are in a certain position. The procedure is as follows. Using the beginning of the first azimuth class as the origin, the azimuth frequencies are grouped into eight 45-degree sectors, then into four pairs of opposite 45-degree sectors, then into four quadrants beginning at the origin, and finally into four quadrants beginning 45 degrees from the origin. For each of these groupings a value of chi-square is calculated, comparing the observed frequencies with a uniform distribution. The origin is then moved to the beginning of the second class and the procedure is repeated. This process is continued until all nine positions of the quadrant have been used. The chi-square values are printed in the form of a table.

Using the computer output, chi-square values were calculated for the polar distribution after regrouping the classes so that the expected frequency in each class exceeded five. The regrouping usually employed combined the three innermost classes into a single group, the three outermost classes into a second group, and adjacent pairs of the six remaining classes into three more groups. No less than five groups could be used, since the number of degrees of freedom of chi-square is one less than the number of groups diminished by the number of parameters of the model distribution that are estimated (Fraser, 1958, p.270). Since in this case three parameters are estimated (k and two coordinates of the mean) the number of degrees of freedom is C-1-3 or C-4 where C is the number of classes. Since the number of degrees of freedom cannot be less than one, C cannot be less than five for this comparison. In the case of the azimuth distribution, the number of estimated parameters is two, since two parameters are required to define the position of the mean and k is not relevant. Hence in this case the minimum number of classes is 1+1+2=4.

COMPUTER PROGRAMS

```
C   PROGRAM  -  POINT DENSITY PLOT 1
      DIMENSION COSA(333),COSB(333),COSG(333),ND(333)
      DIMENSION A(1000), B(1000), G(1000)
      DO [illegible] I=1,[illegible]
      READ(5,500) COSA(I),COSB(I),COSG(I),J
  500 FORMAT(7X,3F11.8,17X,I3)
      IF(I.[illegible].J) [illegible]
    2 CONTINUE
      READ(5,[illegible])KPCA,[illegible]
  501 FORMAT(I[illegible],4X,I[illegible])
      IF(NM.NE.1) GO TO 24
      PCA=KPCA
      TEST=1.-PCA/100.
    [illegible] READ(5,502)[illegible]
  502 FORMAT(I4,I2,I[illegible])
      IF([illegible].LT.[illegible])[illegible] TO [illegible]
      IF(NN.NE.2)GO TO 24
      NCDS=0
    6 READ(5,503)N1,N2,NN,NDIR
  503 FORMAT(I4,I3,I1,I1)
      IF(NN.NE.[illegible])GO TO 24
      IF(N1.LT.0)GO TO 12
      NCDS=NCDS+1
      I=NCDS
      GO TO ([illegible],[illegible],7),[illegible]
    [illegible] IF(NDIR.EQ.0)GO TO 26
      PL=90-N2
      IF(NDIR.NE.1)GO TO 8
      TR=N1+90
      GO TO 11
    8 TR=N1-90
      GO TO 11
    7 PL=90-N2
      TR=N1+180
      IF(TR.LT.360.)GO TO 11
      TR=TR-360.
      GO TO 11
    [illegible] TR=N1
      PL=N2
   11 PL=3.14159265*PL/180.
      TR=3.14159265*TR/180.
      A(I)=COS(TR)*COS(PL)
      B(I)=SIN(TR)*COS(PL)
      G(I)=SIN(PL)
      GO TO 6
   12 CONTINUE
      CDS=NCDS
      DO 15 I=1,333
      DENS=0.
      CSA=COSA(I)
      CSB=COSB(I)
      CSG=COSG(I)
      DO 14 J=1,NCDS
      AV=ABS([illegible](J)+[illegible](J)+[illegible](J)))
      IF(TEST.GT.AV)GO TO 14
      DENS=DENS+1
   14 CONTINUE
   15 ND(I)=[illegible]./[illegible]
      DO 60 K=1,2
      WRITE([illegible])[illegible]
```

```
  600 FORMAT('1PERCENTAGE OF POINTS PER',I2,' PERCENT AREA      STATION N
     1[illegible]',I[illegible],2X,
     2,[illegible]X,I4,'[illegible]OBSERVATIONS')
      GO TO (27,28,29),NZ
   27 WRITE(6,606)
  606 FORMAT(' LINEARS SPECIFIED BY TREND AND PLUNGE')
      GO TO 25
   28 WRITE(6,607)
  607 FORMAT(' POLES OF PLANES: PLANES SPECIFIED BY STRIKE AND DIP')
      GO TO 25
   29 WRITE(6,608)
  608 FORMAT(' POLES OF PLANES: PLANES SPECIFIED BY DIP DIRECTION AND DI
     1P')
   25 WRITE(6,601)  (ND(I), I=1,333)
  601 FORMAT([illegible]
     10X,17I5///10X,17I5///5X,19I5///5X,19I5///5X,19I5/I6,93X,I5//5X,19I
     25/I6,93X,I5//[illegible]
     3//5X,[illegible]///3
     [illegible])
   60 CONTINUE
      GO TO 3
   23 WRITE(6,[illegible])
  602 FORMAT(' ERROR.....POINT COUNTER OUT OF SEQUENCE'//' NUMBERS IN CO
     1LUMNS 58-60 MUST BE IN SEQUENCE FROM 1 TO 333')
      STOP
   24 WRITE(6,603)
  603 FORMAT(' PARAMETER CARD MISSING OR MISPLACED')
      STOP
   26 WRITE(6,605)
  605 FORMAT(50X,'WRONG DATA FORM CODE',//59X,'[illegible]',//[illegible]X,'[illegible]
     1 MISSING ON STRIKE AND DIP DATA CARD')
  100 STOP
      END

C     [illegible]
C                                    [illegible] SECTION 3:  READS PARAMETERS AND
C                                    [illegible] PRELIMINARY CALCULATIONS
      DIMENSION A(1000), B([illegible]), [illegible])
      DIMENSION NOAF(72), NO[illegible], [illegible]
      PI=3.14159
      CF=3.14159/180.
C                                    READ PARAMETER CARD
    3 READ(5,502)NSTN,NN,NG
  502 FORMAT(I4,2X,I2,I3)
      IF(NSTN.LT.0)GO TO 100
      IF(NN.NE.2)GO TO 24
C                                    READ PARAMETER CARD
      READ([illegible]
  501 FORMAT(7X,I1,I4,I3,I3)
      IF(NN.NE.3)GO TO 24
      P=CF*NRAD/12
C                                    CALCULATE DIRECTION COSINES OF REFERENCE LI
      TRR=NTRR
      PLR=NPLR
      TRR=TRR*CF
      PLR=PLR*CF
```

```
      AREF=COS(TRR)*COS(PLR)
      BREF=SIN(TRR)*COS(PLR)
      GREF=SIN(PLR)
C                                    CALCULATE COSINE OF SEMI-ANGLE OF CONE
      RAD=NRAD
      RAD=RAD*CF
      TEST=COS(RAD)
C                                    INITIALIZE SUMS AND COUNTERS
      SA=0.
      SB=0.
      SG=0.
      NTOT=0
      N=0
      DO 30 I=1,12
      NOPE(I)=0
   30 CONTINUE
      DO 33 I=1,72
      NOAE(I)=0
   33 CONTINUE
C                                    PROGRAM SECTION 6:  READS DATA, COUNTS TOTAL
C                                    NUMBER OF OBSERVATIONS IN SAMPLE, NUMBER
C                                    WITHIN CONE, AND COMPUTES SUMS OF DIRECTION
C                                    COSINES OF OBSERVATIONS WITHIN CONE
      READ( ,503)STR,DIP,NDIP
  503 FORMAT(F4.0,F3.0,I2)
      IF(STR.LT.0.)GO TO 14
      NTOT=NTOT+1
C                                    CALCULATE DIRECTION COSINES OF OBSERVATION
C                                    VECTOR (AXIS OR NORMAL TO PLANE)
      NDIR=NDIP+1
      GO TO (10,5,5),NDIR
    5 PL=90.-DIP
      IF(NDIR.NE.2)GO TO 8
      TR=STR+90.
      GO TO 11
    8 TR=STR-90.
      GO TO 11
   10 TR=STR
      PL=DIP
   11 PL=PL*CF
      TR=TR*CF
      AA=COS(TR)*COS(PL)
      BB=SIN(TR)*COS(PL)
      GG=SIN(PL)
C                                    CALCULATE COSINE OF ANGLE BETWEEN REFERENCE
C                                    LINE AND OBSERVATION VECTOR
   13 CALPH=AREF*AA+BREF*BB+GREF*GG
C                                    IF OBSERVATION LIES OUTSIDE CONE GO TO
C                                    NEXT OBSERVATION
      IF(ABS(CALPH).LT.TEST)GO TO 6
      N=N+1
C                                    REVERSE OBSERVATION VECTOR IF NECESSARY
      IF(CALPH.GE.0.)GO TO 12
      AA=-AA
      BB=-BB
      GG=-GG
C                                    INCREMENT SUMS OF DIRECTION COSINES AND RET
C                                    TO READ NEXT DATA CARD
   12 SA=SA+AA
      SB=SB+BB
```

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      SG=SG+GG
      A(N)=AA
      B(N)=BB
      G(N)=GG
      GO TO 6
C                                PROGRAM SECTION 14: CALCULATES AND PRINTS
C                                STATISTICS
   14 WRITE(6,610)NSTN,NG,NTOT,NTRR,NPLR,NRAD,N,SA,SB,SG
  610 FORMAT(//////'1',9X,'STATION NUMBER',I4,'-',I2,5X,'TOTAL SAMPLE:',
     1       I5,' OBSERVATIONS'//
     2  11X,'AXIS OF CONE AT',I4,I3,5X,'SEMI-ANGLE OF CONE:',I3,
     3  ' DEGREES      OBSERVATIONS WITHIN CONE:',I4//
     4  20X,'SA=',F11.4,5X,'SB=',F11.4,5X,'SG=',F11.4//)
   18 R=SQRT(SA*SA+SB*SB+SG*SG)
      AN=N
      RL=100.*R/AN
      IF(SG.GE.0.)GO TO 15
      SA=-SA
      SB=-SB
      SG=-SG
   15 VTR=ATAN(SB/SA)/CF
      VPL=ARSIN(SG/R)/CF
      IF(VTR.GE.0.)GO TO 16
      VTR=VTR+180.
   16 IF(SB.GE.0.)GO TO 17
      VTR=VTR+180.
   17 EXP=1./(AN-1.)
      Q5=20.**(EXP)-1.
      QQ5=1.-Q5*(AN-R)/R
      ALPH5=ARCOS(QQ5)/CF
      Q1=100.**(EXP)-1.
      QQ1=1.-Q1*(AN-R)/R
      ALPH1=ARCOS(QQ1)/CF
      XK=(AN-1.)/(AN-R)
      WRITE(6,600)VTR,VPL,ALPH5,ALPH1,XK,R,RL
  600 FORMAT(20X,'TREND OF MEAN VECTOR =',F6.1,' DEGREES'//
     2  20X,'PLUNGE OF MEAN VECTOR = ',F5.1,' DEGREES'//
     320X,'95 PERCENT CONFIDENCE RADIUS ABOUT MEAN =',F5.1,' DEGREES'//
     420X,'99 PERCENT CONFIDENCE RADIUS ABOUT MEAN =',F5.1,' DEGREES'//
     5  20X,'ESTIMATED PRECISION PARAMETER OF POPULATION =',F7.2,//
     6  20X,'MAGNITUDE OF RESULTANT VECTOR (R) =',F6.1//
     720X,'VECTOR MAGNITUDE L=(R/N)*100 PERCENT =',F5.1,' PERCENT'/////)
C                               CALCULATE POLAR ANGLES BETWEEN OBSERVATIONS
C                               AND RESULTANT, AND AZIMUTH ANGLES REFERRED
C                               TO VERTICAL PLANE THROUGH RESULTANT
      AR=SA/R
      BR=SB/R
      GR=SG/R
      CVPL=SQRT(1.-GR*GR)
      SVTR=BR/CVPL
      CVTR=AR/CVPL
      DO 31 I=1,N
      CPHI=AR*A(I)+BR*B(I)+GR*G(I)
      IF(CPHI.GE.0.)GO TO 49
      A(I)=-A(I)
      B(I)=-B(I)
      G(I)=-G(I)
      CPHI=-CPHI
   49 PHI=ARCOS(CPHI)
      CAZ=A(I)*CVTR+B(I)*SVTR
```

```
      CSC=(CBE-CPHI*CVPL)/(SIN(PHI)*GR)
      IF(CSC.LT.0.9999)GO TO 32
      CSC=0.9999
   32 IF(CSC.GT.-0.9999)GO TO 34
      CSC=-0.9999
   34 C=ARCOS(CSC)
      XYZ=BR*A(I)-AR*B(I)
      IF(XYZ.LT.0.)GO TO 27
      C=2.*PI-C
C                                     INCREMENT CLASS COUNTS
   27 J=36.*C/PI+1.
      NOAF(J)=NOAF(J)+1
      J=PHI/P+1.
      NOPF(J)=NOPF(J)+1
   31 CONTINUE
C                                     CALCULATE POLAR CLASS LIMITS, EXPECTED
C                                     FREQUENCY IN POLAR CLASSES
      QX=1.
      DO 29 I=1,12
      F=I
      PLIM(I)=P*F/CF
      Q1=-XK+XK*COS(P*F)
      Q=2.71828**Q1
      EPF(I)=AN*(QX-Q)
      QX=Q
   29 CONTINUE
C                                     PRINT CLASS LIMITS AND OBSERVED AND EXPECTED
C                                     FREQUENCIES
      WRITE(6,648)  (PLIM(I), I=1,12)
  648 FORMAT(' POLAR CLASS LIMITS      ',12F8.1/)
      WRITE(6,647)  (EPF(I), I=1,12)
  647 FORMAT(' EXPECTED POLAR FREQ     ',12F8.0/)
      WRITE(6,646)  (NOPF(I), I=1,12)
  646 FORMAT(' OBSERVED POLAR FREQ     ',12I8///)
      WRITE(6,645)  (NOAF(I), I=1,72)
  645 FORMAT(' OBSERVED AZIMUTH FREQ ',18I5)
C                                     CALCULATE CHI-SQUARE ON AZIMUTH DISTRIBUTION
C                                     USING DIFFERENT PATTERNS OF CLASSES AND
C                                     NINE DIFFERENT ORIGINS AT 5-DEG INTERVALS
      WRITE(6,700)
  700 FORMAT(///10X,'CHI-SQUARE VALUES FOR AZIMUTH DISTRIBUTION:'//
     1  22X,'180-DEGREE DISTRIBUTION',22X,'I',11X,'360-DEGREE DISTRIBUTI
     [illegible]
     [illegible]
     [illegible]
      [illegible]
   36 S1=0
      S2=0
      S3=0
      S4=0
      S5=0
      S6=0
      S7=0
      S8=0
      DO 37 I=1,9
   37 S1=S1+NOAF(I)
      DO 38 I=10,18
   38 S2=S2+NOAF(I)
      DO 39 I=19,27
   39 S3=S3+NOAF(I)
```

```
      DO 40 I=28,36
   40 S4=S4+NOAF(I)
      DO 41 I=37,45
   41 S5=S5+NOAF(I)
      DO 42 I=46,54
   42 S6=S6+NOAF(I)
      DO 43 I=55,63
   43 S7=S7+NOAF(I)
      DO 44 I=64,72
   44 S8=S8+NOAF(I)
   46 IF(N.LT.10)GO TO 48
      E=N/2
      CS(1)=((S1+S2+[illegible]-E)**2+([illegible]+S4+S6+S[illegible]-E)**2)/E
      CS(2)=((S1+S2+S5+S6-E)**2+(S3+S4+S7+S8-E)**2)/E
      CS(3)=((S2+S3+S6+S7-E)**2+(S8+S1+S4+S5-E)**2)/E
      ICS=3
      IF(N.LT.20)GO TO 45
      E=N/4
      CS(4)=((S1+S5-E)**2+(S2+S6-E)**2+(S3+S7-E)**2+(S4+S8-E)**2)/E
      CS(5)=((S1+S2-E)**2+(S3+S4-E)**2+(S5+S6-E)**2+(S7+S8-E)**2)/E
      CS(6)=((S2+S3-E)**2+(S4+S5-E)**2+(S6+S7-E)**2+(S8+S1-E)**2)/E
      ICS=6
      IF(N.LT.40)GO TO 45
      E=N/8
      CS(7)=((S1-E)**2+(S2-E)**2+(S3-E)**2+(S4-E)**2+(S5-E)**2+([illegible]-E)**2
     1 +(S7-E)**2+(S[illegible]-E)**2)/E
      ICS=7
   45 WRITE(6,643) (CS(I),I=1,ICS)
  643 FORMAT(1X,F11.2,7X,2('I',F12.2,F8.2,7X,'I',F12.2,7X))
      IT=IT+1
      IF(IT.GE.9)GO TO 3
      NOAF1=NOAF(1)
      DO 47 I=1,71
   47 NOAF(I)=NOAF(I+1)
      NOAF(72)=NOAF1
      GO TO 3[illegible]
   48 WRITE(6,641)
  641 FORMAT(10X,'SAMPLE TOO SMALL TO CALCULATE CHI-SQUARE')
      GO TO 3
C                                    ERROR MESSAGE
   24 WRITE(6,603)
  603 FORMAT(' PARAMETER CARD MISSING OR MISPLACED')
  100 STOP
      END


C  PROGRAM  -  VECTOR MAGNITUDES 1
      DIMENSION [illegible]
      DIMENSION A(1000), B(1000), G(1000)
C     READ AND STORE DIRECTION COSINES OF NORMALS TO REFERENCE PLANES,
C     CHECKING THAT CARDS ARE IN SEQUENCE
      PI=3.1415926
      DO 2 I=1,333
      READ([illegible]) [illegible](I),[illegible](I),[illegible](I),J
  [illegible] FORMAT(7X,3F11.8,17X,I3)
      IF(I.NE.J)GO TO 23
    2 CONTINUE
```

```
C       READ PARAMETER CARD
      [illegible]
  [illegible] FORMAT(I4,I2,I2,I3)
        IF(NSTN.LT.0)GO TO 100
        IF(NN.NE.2)GO TO 24
        NZ=NZ+1
C       READ DATA, COUNT NUMBER OF OBSERVATIONS, CALCULATE DIRECTION COSIN
C       ES AND STORE
        NCDS=0
      4 READ(5,503)STR,DIP,NDIP
    503 [illegible]
        IF(STR.LT.0.)GO TO 5
        NCDS=NCDS+1
        GO TO (9,10),NZ
      9 [illegible] TO 26
        PL=90.-DIP
        IF(NDIR.NE.1)GO TO 11
        TR=STR+90.
        GO TO [illegible]
     11 TR=STR-90.
        GO TO 12
     10 TR=STR
        PL=DIP
     12 PL=PI*PL/180.
        TR=PI*TR/180.
        I=NCDS
        A(I)=COS(TR)*COS(PL)
        B(I)=SIN(TR)*COS(PL)
        G(I)=SIN(PL)
        GO TO 4
      5 CDS=NCDS
C       LOOP TO CONSIDER EACH REFERENCE PLANE
        [illegible]
        CSA=COSA(I)
        CSB=COSB(I)
        CSG=COSG(I)
C       INITIALIZE SUMS OF DIRECTION COSINES
     13 SA=0.
        SB=0.
        SG=0.
C       LOOP TO CONSIDER EACH OBSERVATION
        DO 7 J=1,NCDS
C       CALCULATE COSINE OF ANGLE BETWEEN NORMAL TO REFERENCE PLANE AND
C       OBSERVATION VECTOR
        CALPH=CSA*A(J)+CSB*B(J)+CSG*G(J)
C       ADJUST OBSERVATION VECTOR TO LOWER SIDE OF REFERENCE PLANE
        IF(CALPH.GE.0.)GO TO 8
        SA=SA-A(J)
        SB=SB-B(J)
        SG=SG-G(J)
        GO TO 7
C       INCREMENT SUMS OF DIRECTION COSINES
      8 SA=SA+A(J)
        SB=SB+B(J)
        SG=SG+G(J)
      7 CONTINUE
C       CALCULATE MAGNITUDE OF RESULTANT AND EXPRESS AS PERCENT
     14 R=SQRT(SA**2+SB**2+SG**2)
        NL(I)=100.*R/CDS + 0.5
      6 CONTINUE
```

```
C       PRINT RESULTS AND RETURN FOR NEXT DATA SET
        WRITE(6,600)NSTN,NG,NCDS
  600 FORMAT('1VECTOR MAGNITUDES (100*P/N) FOR 333 POSITIONS OF REFERENC
     1E PLANE      STATION NUMBER',I4,'-',I2,10X,I4,' OBSERVATIONS',/' RI
     2GHT HAND DIGIT IS AT POLE OF REFERENCE PLANE')
        WRITE(6,601) (NL(I),I=1,333)
  601 FORMAT(50X,I5,/39X,I5,I6,I10,I6//30X,9I5///20X,13I5///15X,15I5///1
     1[illegible]
     2[illegible]
     3//5X,19I5///10X,17I5///10X,17I5///15X,15I5///20X,13I5///30X,9I5//3
     4[illegible]
        GO TO 3
C       ERROR MESSAGES
   23 WRITE(6,602)
  602 FORMAT(' ERROR.....COUNTER CARDS OUT OF SEQUENCE'//' NUMBERS IN CO
     1LUMNS 58-60 MUST BE IN SEQUENCE FROM 1 TO 333')
        STOP
   24 WRITE(6,603)
  603 FORMAT(' PARAMETER CARD MISSING OR MISPLACED - SEE INSTRUCTIONS')
        STOP
   26 WRITE(6,605)
  605 FORMAT(50X,'WRONG DATA FORM CODE',//59X,'OR',//35X,'DIRECTION CODE
     1 MISSING ON STRIKE AND DIP DATA CARD')
  100 STOP
        END

C MODIFIED POINT DENSITY 4                    MFPD4
C
C PART 1  -  DIMENSION VARIABLES AND DEFINE CONSTANTS
        DIMENSION COSA(333), COSB(333), COSG(334), ND(334)
        DIMENSION A(1000), B(1000), G(1000), VOL(100)
        DIMENSION R(45), DPRU(45), AR(45)
        PI=3.1415926
        E=2.71828
        CF=PI/180.
        HD=0.01*CF
C
C
C
C PART 2  -
C READ AND STORE DIRECTION COSINES OF COUNTING LOCATIONS , CHECKING THAT
C COUNTER CARDS ARE IN SEQUENCE
        DO 2 I=1,333
        READ(5,500)COSA(I),COSB(I),COSG(I),J
  500 FORMAT(7X,3F11.8,17X,I3)
        IF(I.NE.J)GO TO 23
    2 CONTINUE
C
C
C
C PART 3  -  READ PARAMETER CARD 1, PERFORM CHECKS, AND CHANGE
C PARAMETERS TO FLOATING POINT
        READ(5,501)KPCA,NDISP,NN,KP,KOPY
  501 FORMAT(I3,I2,I3,I4,I2)
        IF(NN.NE.1)GO TO 24
        PCA=KPCA
```

```
      XK=XP
C
C
C
C PART 4  -  CALCULATION OF PROBABILITY DISTRIBUTION
      QX=0.
      SDPR=0.
      I=0
   16 I=I+1
      F=I
      U(I)=F*CF
      Q=1.-COS(U(I))
      EXP1=-XK*QX
      EXP2=-XK*Q
      DPR=E**EXP1-E**EXP2
      SDPR=SDPR+DPR
      AR(I)=2.*PI*Q
      ARI=AR(I)
      IF(I.EQ.1)GO TO 27
      DA=AR(I)-AR(I-1)
      PRU=DPR/DA
      DPRU(I-1)=PRUX-PRU
      IF(NDISP.EQ.0)GO TO 41
      WRITE(6,610)I,DPR,SDPR,ARI,DA,PRU
  610 FORMAT(5X,'I=',I3,5X,'DPR=',F6.4,10X,'SDPR=',F6.4,
     1       10X,'A=',F10.5,10X,'DA=',F10.5,10X,'PRU=',F10.5)
   41 IF(SDPR.GT. .9999)GO TO 17
      GO TO 22
   27 PRU=DPR/AR(I)
      IF(NDISP.EQ.0)GO TO 22
      WRITE(6,612)I,DPR,SDPR,ARI,PRU
  612 FORMAT(5X,'I=',I3,5X,'DPR=',F6.4,10X,'SDPR=',F6.4,
     1       10X,'A=',F10.5,31X,'PRU=',F10.5)
   22 QX=Q
      PRUX=PRU
      GO TO 16
   17 DPRU(I)=PRU
      IMAX=I
      IF(NDISP.EQ.0)GO TO 42
      WRITE(6,609)    (DPRU(I),    I=1,IMAX)
  609 FORMAT(/10X,'DPRU IN ORDER OF INCREASING I'/10X,10F10.4)
C
C
C
C PART 5  -  CALCULATION OF AREA AND RADIUS OF COUNTING CIRCLE
   42 ACC=2.*PI*PCA/100.
      CSC=1.-PCA/100.
      C=ARCOS(CSC)
      CD=C/CF
      ICD=CD+0.5
      IF(NDISP.EQ.0)GO TO 43
      WRITE(6,608) CD
  608 FORMAT(/10X,'RADIUS OF COUNTING CIRCLE=',F5.2///)
C
C
C
C PART 6  -  COMPUTATION OF THE ARRAY 'VOL'
      NERO=0
   43 IDD2=ICD+IMAX
      IDD1=ICD-IMAX
```

```
      IF(IDD1.GT.0)GO TO 44
      IDD1=0
      GO TO 45
   44 DO 46 J=1,IDD1
      VOL(J)=1.
   46 CONTINUE
   45 DO 47 J=IDD1,IDD2
      D=J*CF
      CSD=COS(D)
      VOLUM=0.
      I=IMAX
      F=I
   48 RA=R(I)
      IF(D.LE.ABS(C-RA)+HD)GO TO 50
      IF(D.GT.C+RA-HD)GO TO 49
      CSR=COS(RA)
      CAL=(CSR-CSC*CSD)/(SIN(C)*SIN(D))
      CBE=(CSC-CSR*CSD)/(SIN(RA)*SIN(D))
      CGA=(CSD-CSC*CSR)/(SIN(C)*SIN(RA))
      IF(ABS(CAL).GT.1.)GO TO 36
      IF(ABS(CBE).GT.1.)GO TO 36
      IF(ABS(CGA).GT.1.)GO TO 36
   33 ALPHA=ARCOS(CAL)
      BETA=ARCOS(CBE)
      GAMMA=ARCOS(CGA)
      AREA=2.*(PI-GAMMA-ALPHA*CSC-BETA*CSR)
      GO TO 52
   36 WRITE(6,606)[illegible]
  606 FORMAT(10X,'*********************INVALID COSINE*********************'/
     2  15X,'RADIUS OF [illegible] CIRCLE =',[illegible],' DEG'/
     3  15X,'DISTANCE IN TENTHS OF DEGREE =',I5/
     4  15X,'IMAX =',I4,20X,'I=',I4/
     5  [illegible]///)
      AREA=0.
      NERO=1
      GO TO 52
   50 IF(AR(I).GE.ACC)GO TO 51
      AREA=AR(I)
      GO TO 52
   51 AREA=ACC
   52 VOLUM=VOLUM+AREA*DPRU(I)
      I=I-1
      IF(I.GT.0)GO TO 48
   49 VOL(J+1)=VOLUM
   47 CONTINUE
      IF(NERO.EQ.1)GO TO 100
      IDD3=IDD2+1
      IF(NDISP.EQ.0)GO TO 3
      DO 55 I=1,IDD3
      IDIST=I-1
      WRITE(6,613)IDIST,VOL(I)
  613 FORMAT(5X,'DISTANCE =',I5,' DEGREES',20X,'VOLUME =',F6.4)
   55 CONTINUE
C
C
C
C PART 7 - READ PARAMETER CARD 2, READ DATA, CALCULATE DIRECTION
C COSINES AND STORE, COUNT DATA CARDS
    3 READ(5,502)NSTN,NZ,NN,NG
  502 FORMAT(I4,I2,I2,I3)
```

```
      IF(NSTN.LT.0)GO TO 100
      IF(NN.NE.2)GO TO 24
      NCDS=0
    6 READ(5,503)N1,N2,NN,NDIR
  503 FORMAT(I4,I3,I1,I1)
      IF(NN.NE.0)GO TO 24
      IF(N1.LT.0)GO TO 12
      NCDS=NCDS+1
      I=NCDS
      GO TO (9,5,7),N7
    5 IF(NDIR.EQ.0)GO TO 26
      PL=90-N2
      IF(NDIR.NE.1)GO TO 8
      TR=N1+90
      GO TO 11
    8 TR=N1-90
      GO TO 11
    7 PL=90-N2
      TR=N1+180
      IF(TR.LT.360.)GO TO 11
      TR=TR-360.
      GO TO 11
    9 TR=N1
      PL=0.
   11 PL=PL*CF
      TR=TR*CF
      A(I)=COS(TR)*COS(PL)
      B(I)=SIN(TR)*COS(PL)
      G(I)=SIN(PL)
      GO TO 6
   12 CDS=NCDS
C
C
C
C PART 8  -  COUNTING PROCEDURE
      DDMAX=IDD2+0.5
      DMAX=DDMAX*CF
      CSDM=COS(DMAX)
      DO 53 L=1,333
      COUNT=0.
      CSA=COSA(L)
      CSB=COSB(L)
      CSG=COSG(L)
      DO 54 J=1,NCDS
      CSD=ABS(CSA*A(J)+CSB*B(J)+CSG*G(J))
      IF(CSD.LE.CSDM)GO TO 54
      K=ARCOS(CSD)/CF+1.5
      COUNT=COUNT+VOL(K)
   54 CONTINUE
      ND(L)=COUNT*100./CDS
   53 CONTINUE
C
C
C
C PART 9  -  WRITE RESULTS AND RETURN FOR NEXT DATA SET
      DO 40 J=1,KOPY
      WRITE(6,600)PERCA,NSTN,NG,NCDS,KP
  600 FORMAT('1MODIFIED POLE COUNT PER',I2,' PERCENT AREA      STATION NU
     1MBER',I4,'-',I2,I9,' OBSERVATIONS, PRECISION PARAMETER (K) =',I4,/
     2       ' NUMBERS TRUNCATED, NOT ROUNDED  NFPD4')
```

```
      WRITE(6,601)   (ND(I), I=1,333)
  601 FORMAT(50X,I5,/39X,I5,I6,I10,I6//30X,9I5///20X,13I5///15X,15I5///1
     10X,17I5///10X,17I5///5X,19I5///5X,19I5///5X,19I5/I6,93X,I5//5X,19I
     25/I6,93X,I5//21I5//I6,93X,I5/5X,19I5//I6,93X,I5/5X,19I5///5X,19I5/
     3//5X,19I5///1[illegible]
     49X,I5,I6,I10,I6,/50X,I5)
   40 CONTINUE
      GO TO 3
C
C
C
C PART 10   -   ERROR MESSAGES
   23 WRITE(6,602)
  602 FORMAT(' ERROR.....POINT COUNTER OUT OF SEQUENCE'//' NUMBERS IN CO
     1LUMNS 58-60 MUST BE IN SEQUENCE FROM 1 TO 333')
      STOP
   24 WRITE(6,603)
  603 FORMAT(' PARAMETER CARD MISSING OR MISPLACED - SEE INSTRUCTIONS')
      STOP
   26 WRITE(6,605)
  605 FORMAT(50X,'WRONG DATA FORM CODE',//59X,'OR',//35X,'DIRECTION CODE
     1 MISSING ON STRIKE AND DIP DATA CARD')
  100 STOP
      END
```

CENTRAL STORES HF-585

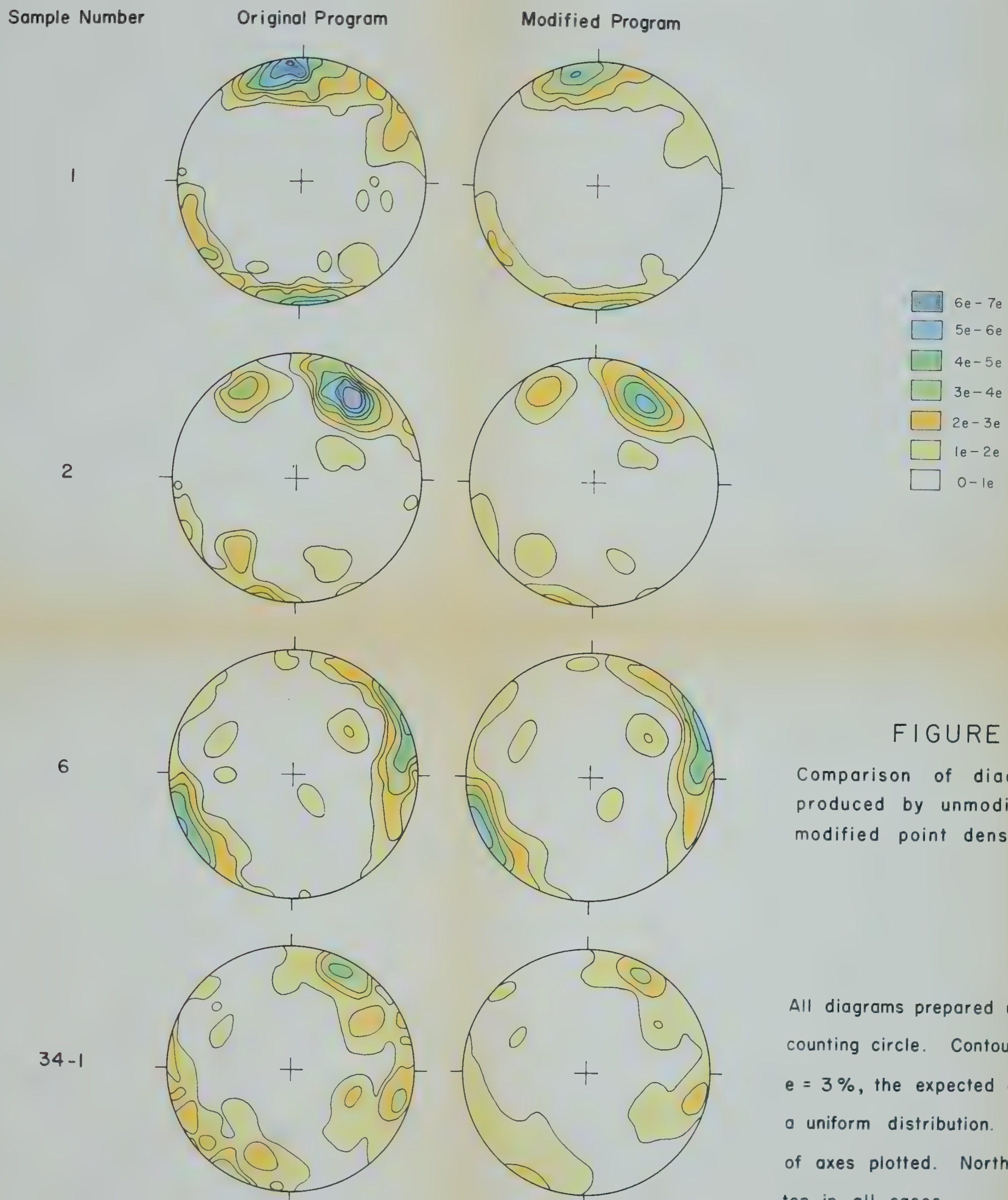


FIGURE 3

Comparison of diagrams produced by unmodified and modified point density programs.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

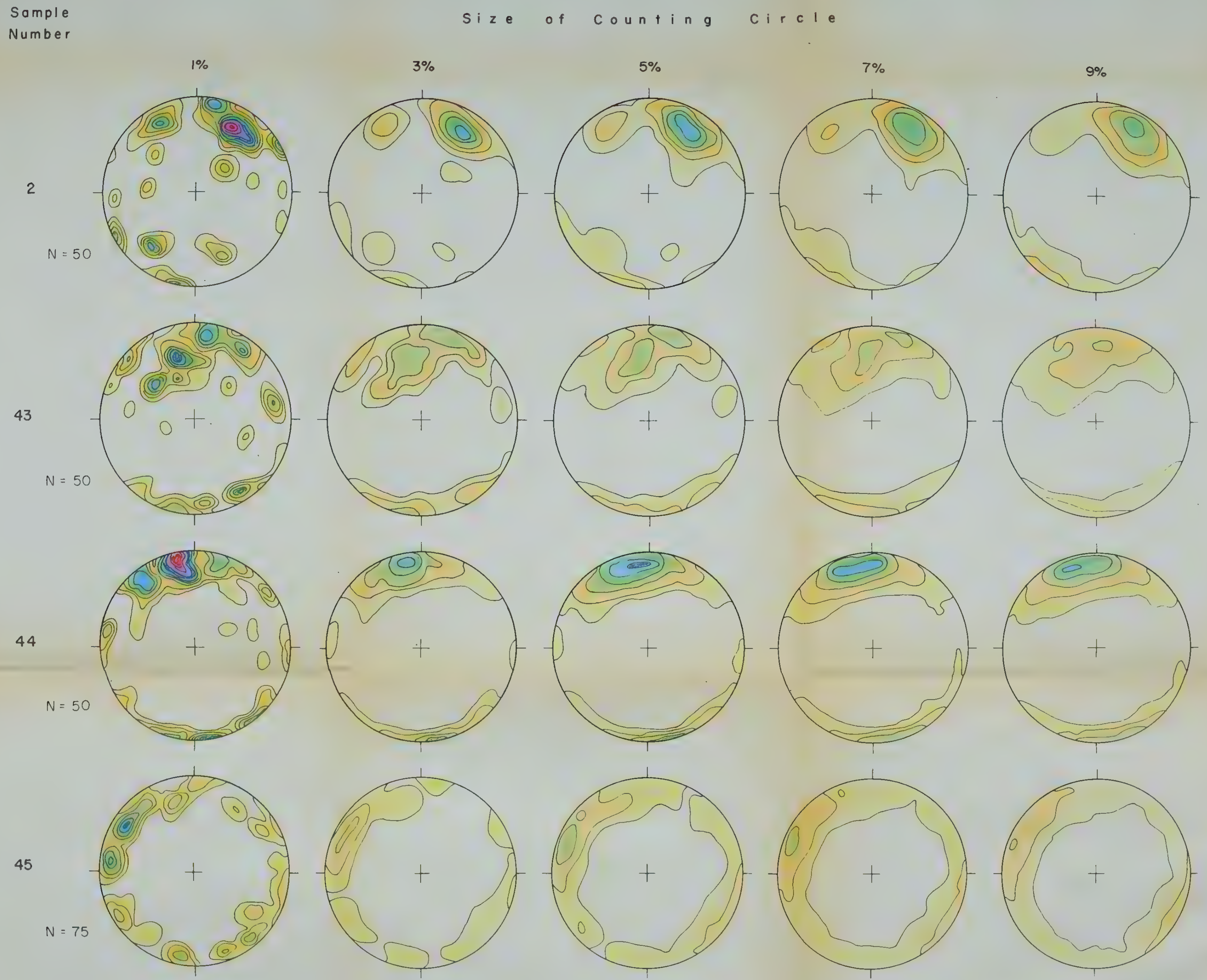


FIGURE 4.

Effect on point density diagrams of change in size of counting circle.

Contour interval = e, where e is the expected density in the case of a sample from a uniform distribution, and is equal to the per cent size of the counting circle. North at the top in all cases.

Key to Fabric Diagrams

11e - 12e	7e - 8e	3e - 4e
10e - 11e	6e - 7e	2e - 3e
9e - 10e	5e - 6e	1e - 2e
8e - 9e	4e - 5e	0 - 1e

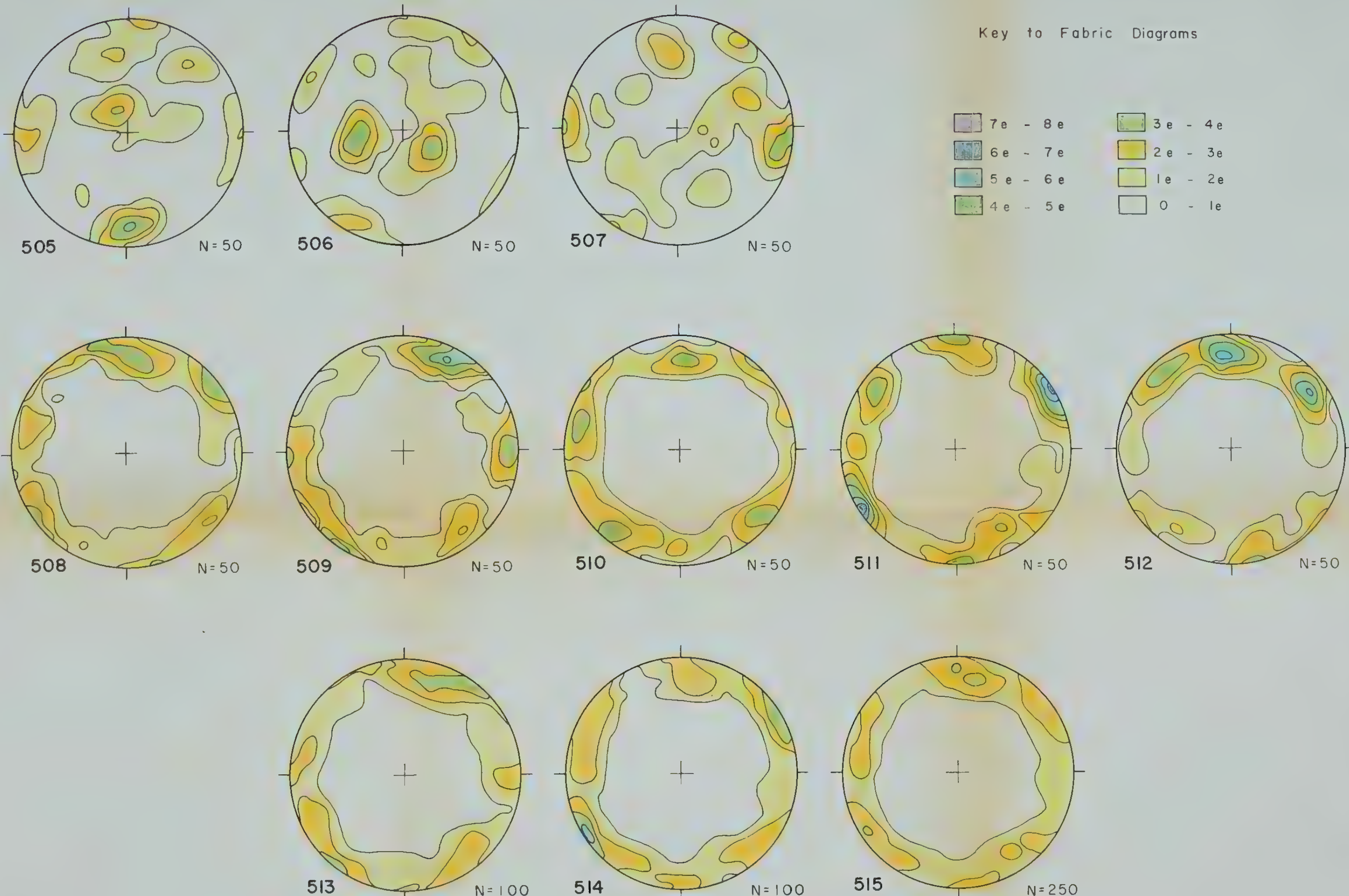


FIGURE 5. Diagrams of samples from a uniform distribution (505-507) and a hypothetical girdle distribution (508-515).

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

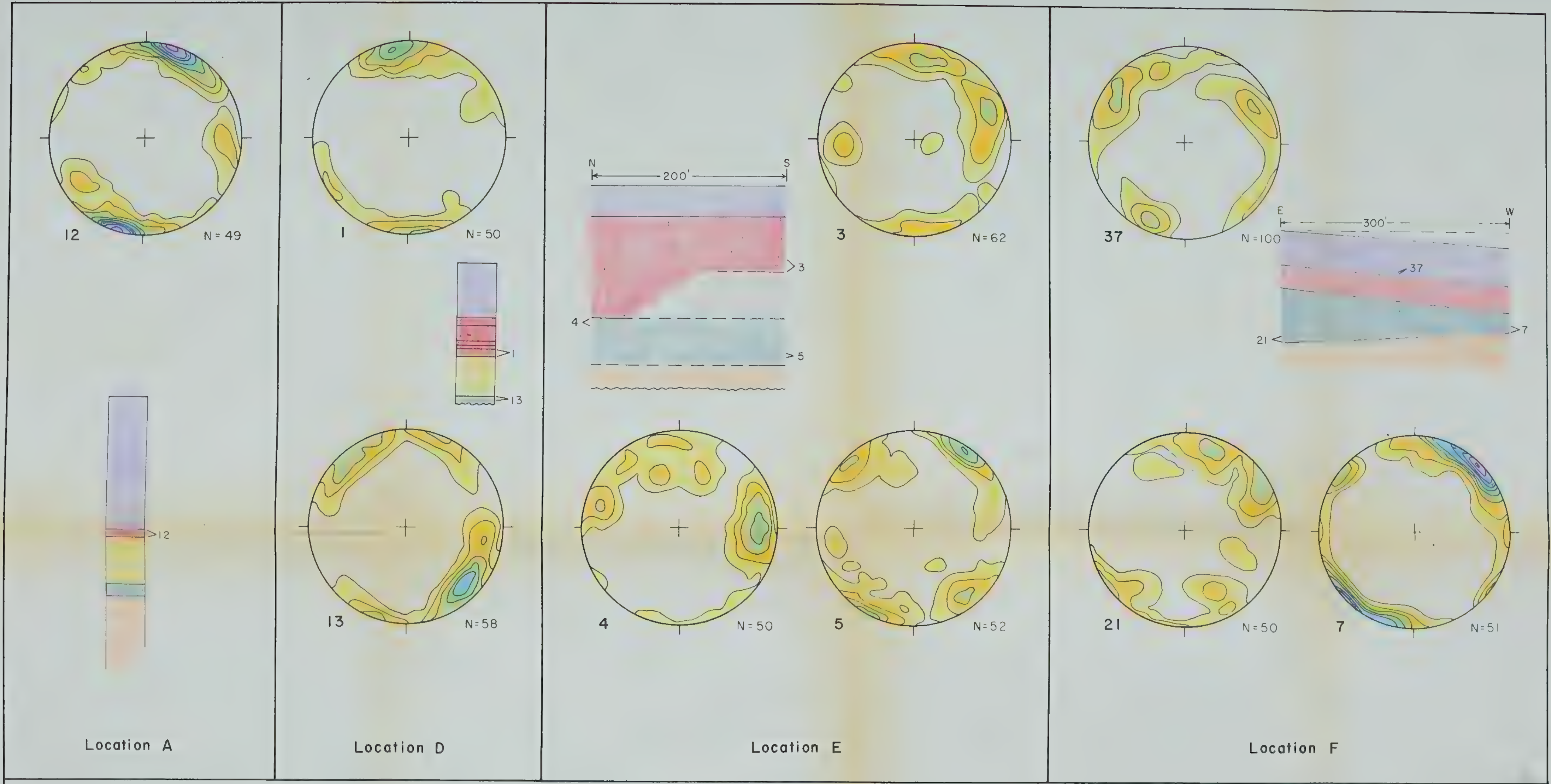


FIGURE 6. Fabric diagrams, Locations A, D, E and F, except upper till at Location F.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

Key to Fabric Diagrams

9e–10e
8e–9e
7e–8e
6e–7e
5e–6e
4e–5e
3e–4e
2e–3e
1e–2e
0–1e

Key to Stratigraphic Diagrams

Stratigraphic Units

Lake Edmonton Sediments
Upper Till
Tofield Sand
Lower Till
Saskatchewan Gravels
Edmonton Formation

General

Ground Surface
Lower Limit of Exposure
Upper Limit of Exposure if Not Ground Surface
Diagram Not Extending to Base of Exposure

Vertical Scale for All Diagrams 1". 20'

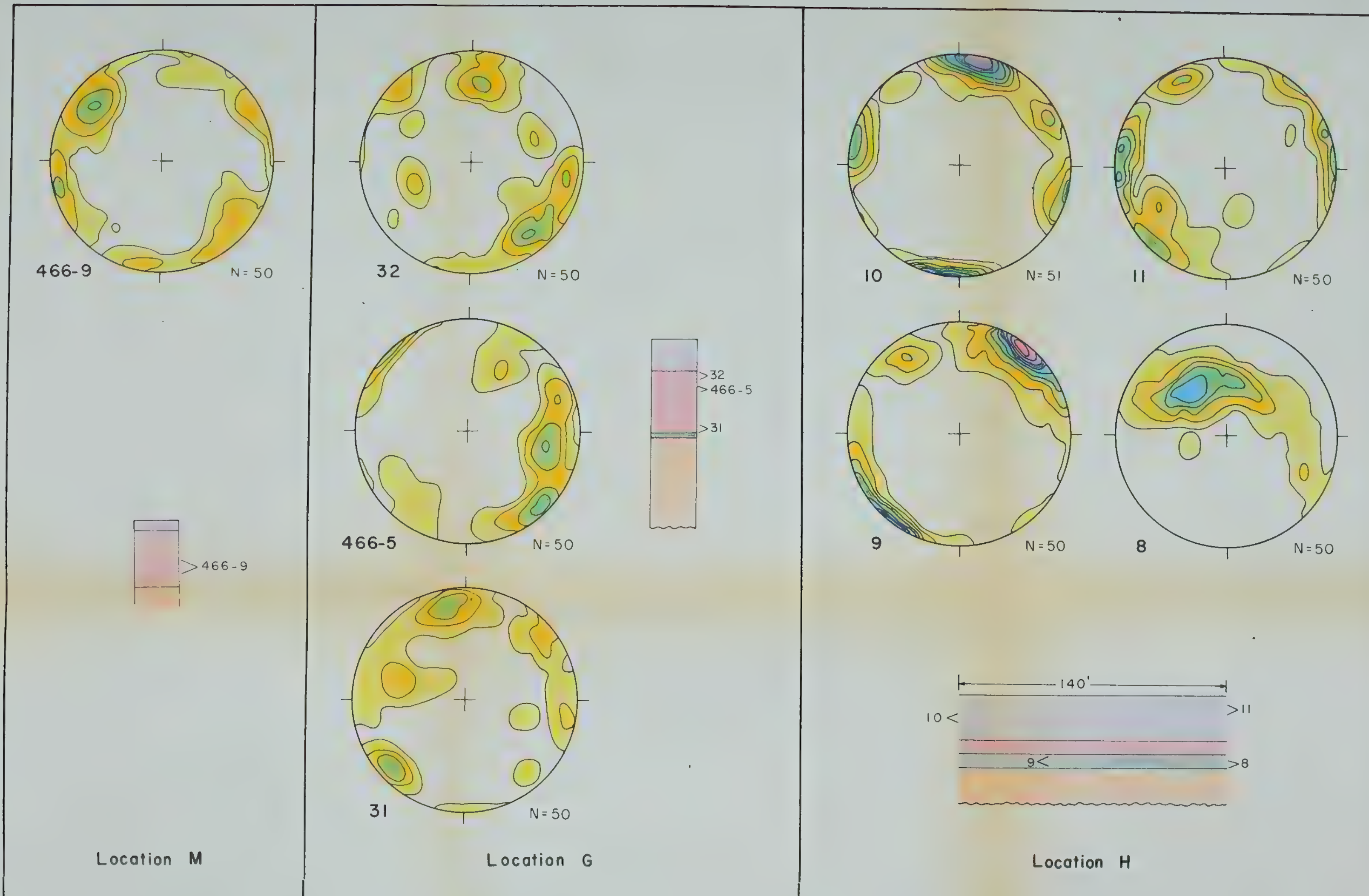


FIGURE 7. Fabric diagrams, Locations M, G and H.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

Key to Fabric Diagrams

- 9e-10e
- 8e-9e
- 7e-8e
- 6e-7e
- 5e-6e
- 4e-5e
- 3e-4e
- 2e-3e
- 1e-2e
- 0-1e

Key to Stratigraphic Diagrams

Stratigraphic Units

- Lake Edmonton Sediments
- Upper Till
- Tofield Sand
- Lower Till
- Saskatchewan Gravels
- Edmonton Formation

General

- Ground Surface
- Lower Limit of Exposure
- Upper Limit of Exposure if Not Ground Surface
- Diagram Not Extending to Base of Exposure

Vertical Scale for All Diagrams 1" : 20'

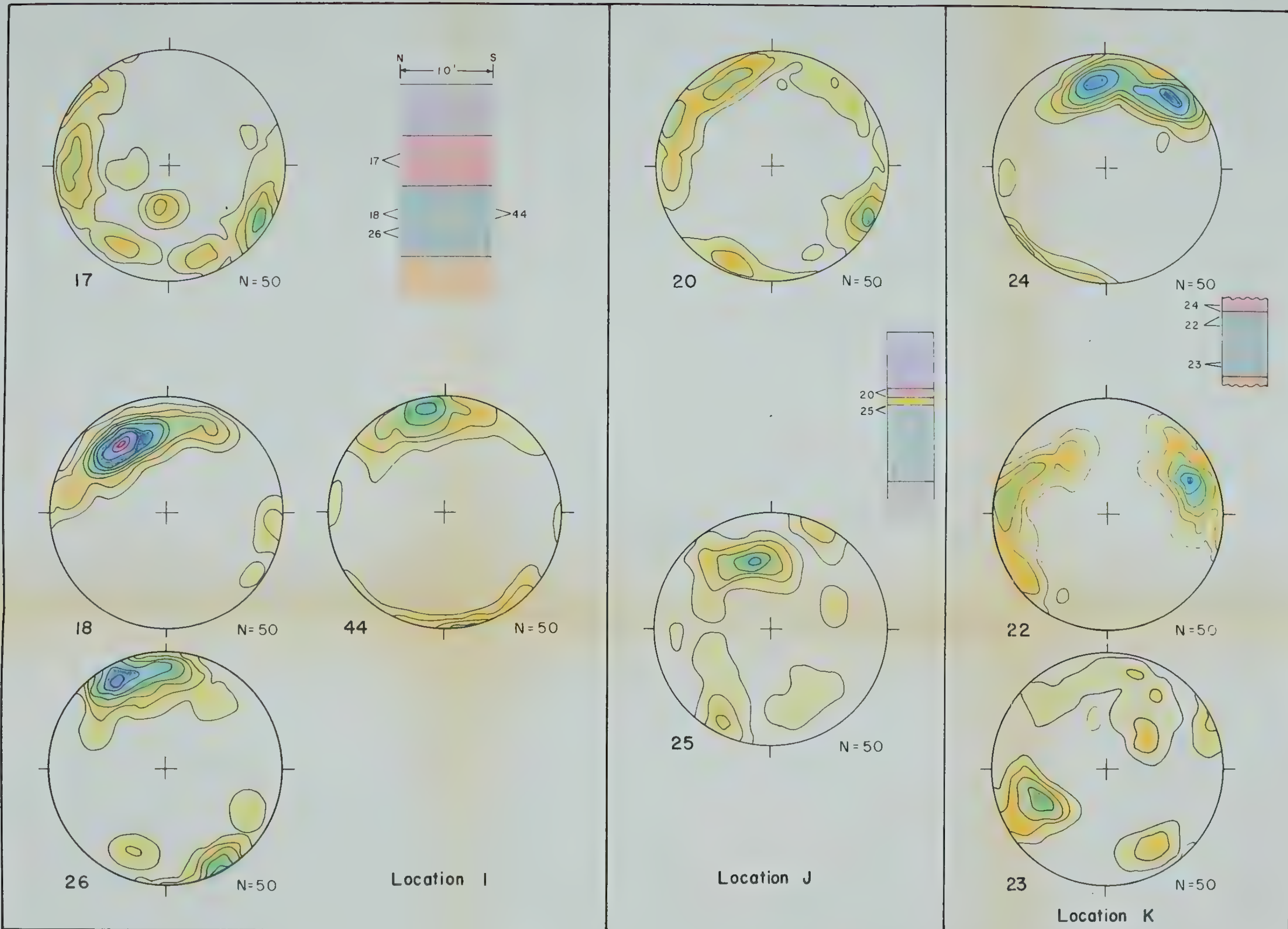


FIGURE 9. Fabric diagrams, Locations I, J and K.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution.

N = number of axes plotted. North at the top in all cases.

Key to Fabric Diagrams

- 9e - 10e
- 8e - 9e
- 7e - 8e
- 6e - 7e
- 5e - 6e
- 4e - 5e
- 3e - 4e
- 2e - 3e
- 1e - 2e
- 0 - 1e

Key to Stratigraphic Diagrams

Stratigraphic Units

- Lake Edmonton Sediments
- Upper Till
- Tofield Sand
- Lower Till
- Saskatchewan Gravels
- Edmonton Formation

General

- Ground Surface
- Lower Limit of Exposure
- Upper Limit of Exposure if Not Ground Surface
- Diagram Not Extending to Base of Exposure

Vertical Scale for All Diagrams 1" = 20'

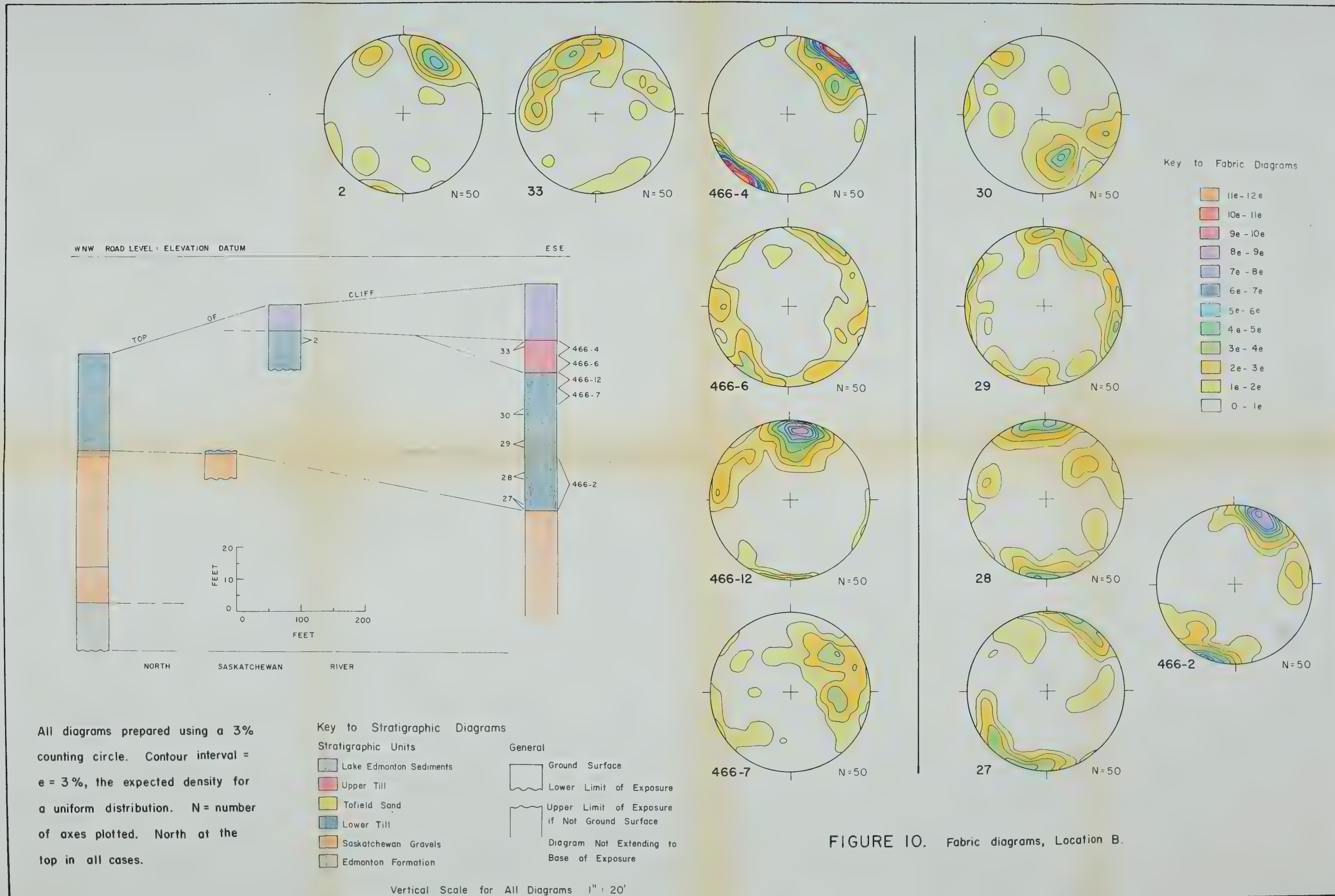
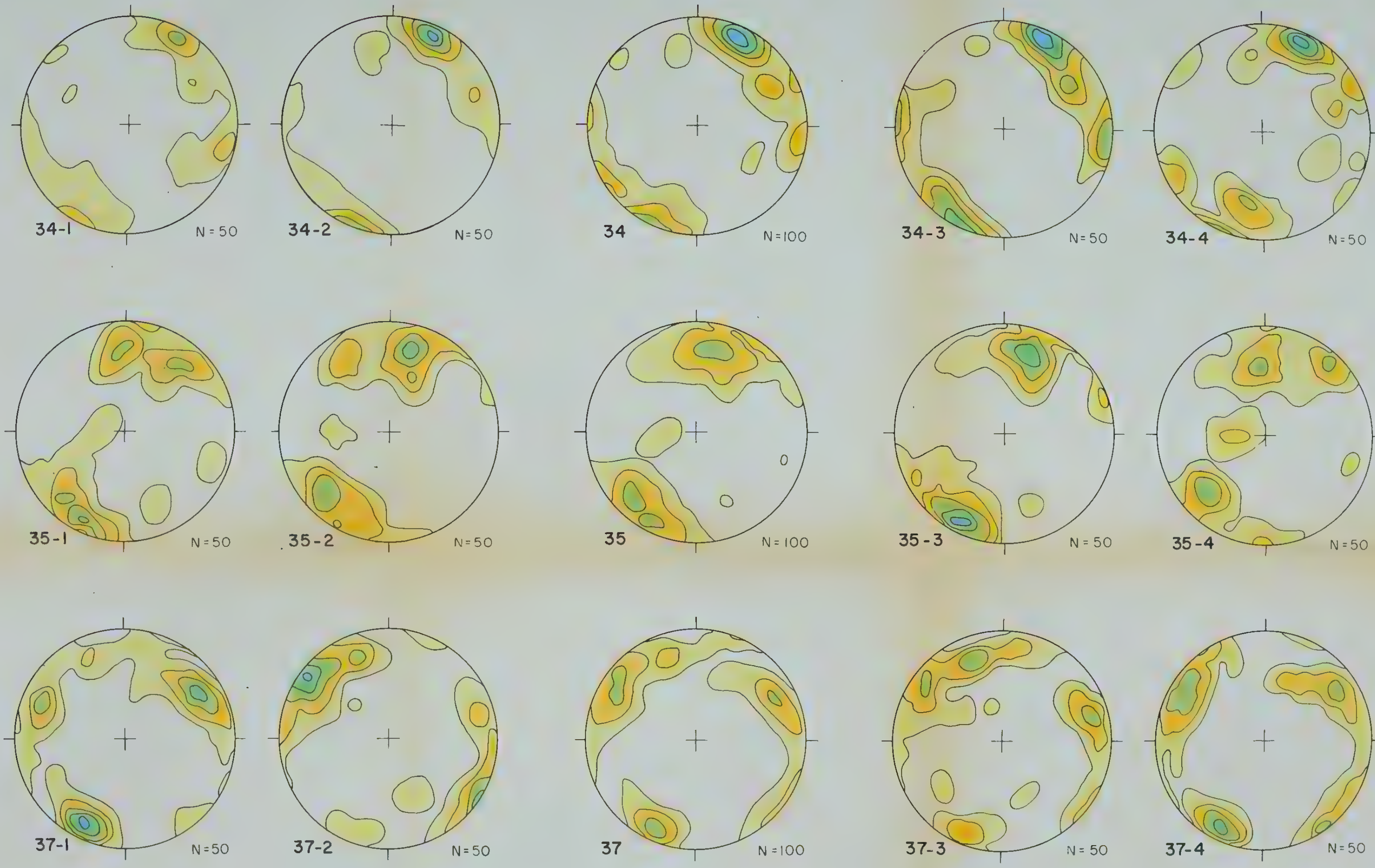


FIGURE 10. Fabric diagrams, Location B.



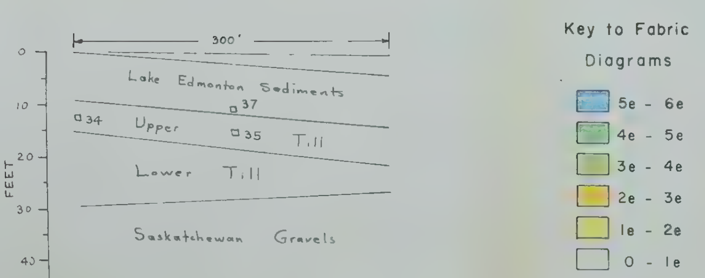
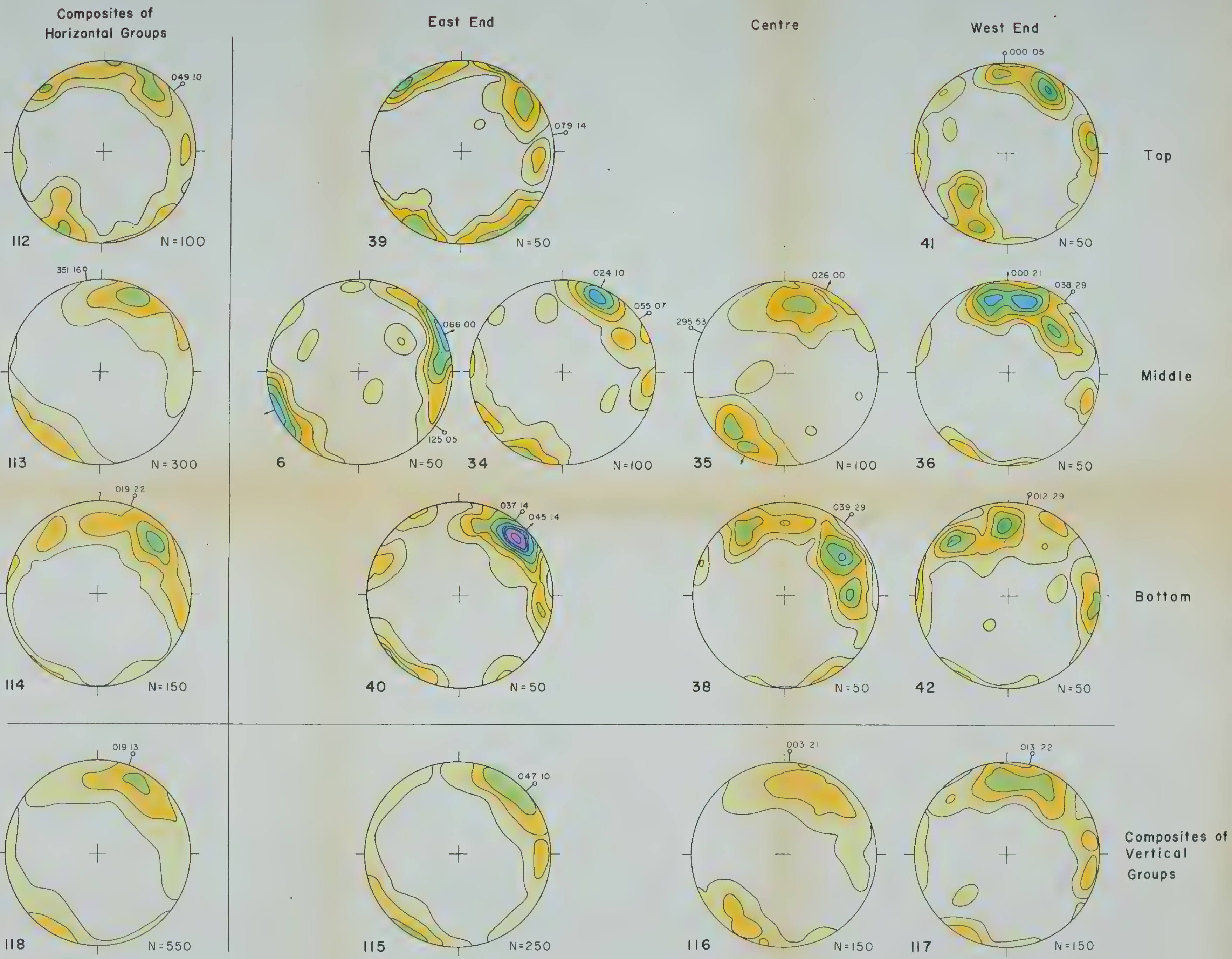


FIGURE 11.

Reproducibility of 50-axis samples from sites 34, 35 and 37 at Location F.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.



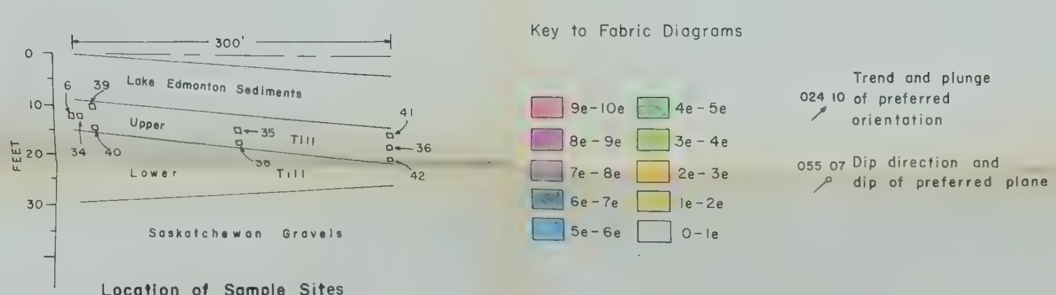
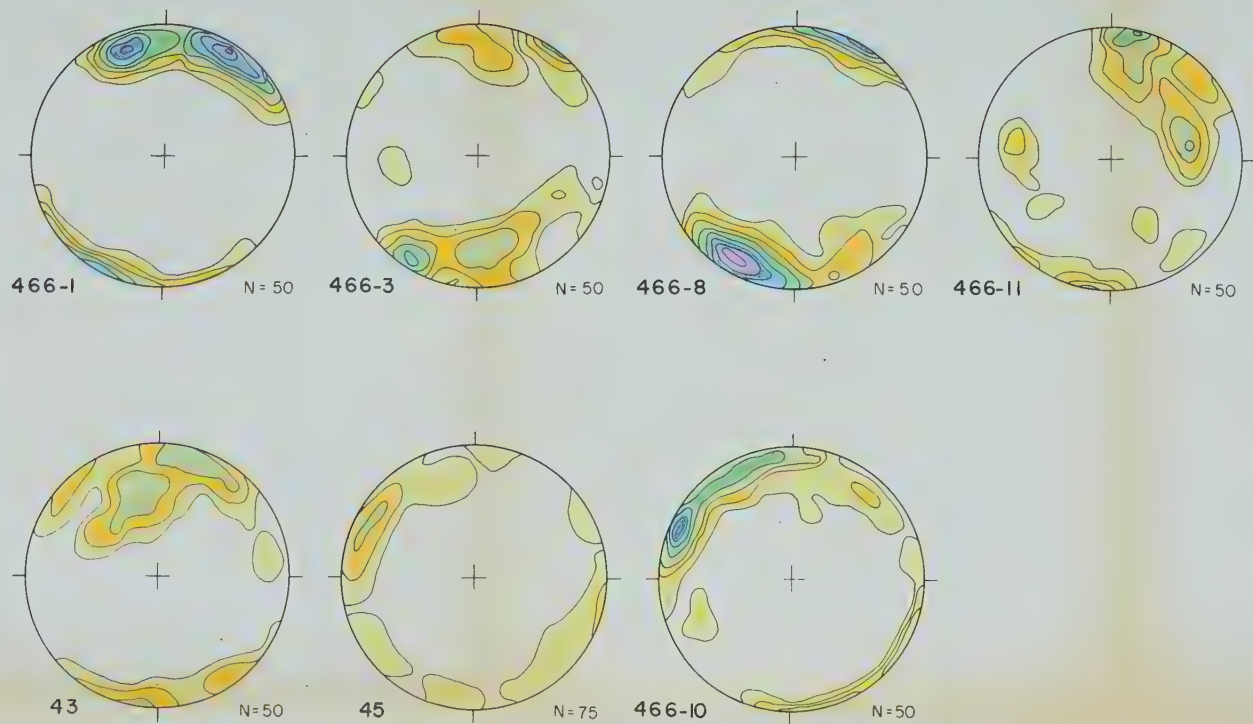
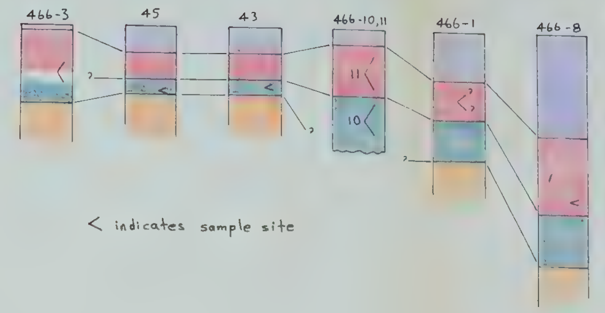


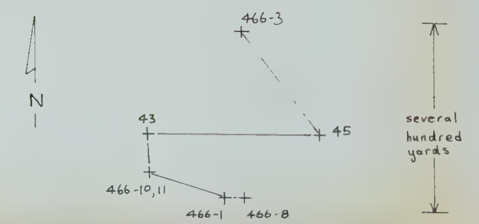
FIGURE 12

Fabric variability in upper till at Location F.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.







Sketch (not to scale) of sample locations showing line of stratigraphic section

Key to Fabric Diagrams

- 9e-10e
- 8e-9e
- 7e-8e
- 6e-7e
- 5e-6e
- 4e-5e
- 3e-4e
- 2e-3e
- 1e-2e
- 0-1e

Key to Stratigraphic Diagrams

Stratigraphic Units

- Lake Edmonton Sediments
- Upper Till
- Tofield Sand
- Lower Till
- Saskatchewan Gravels
- Edmonton Formation

General

- Ground Surface
- Lower Limit of Exposure
- Upper Limit of Exposure if Not Ground Surface
- Diagram Not Extending to Base of Exposure

Vertical Scale 1": 20'

FIGURE 13 Fabric diagrams, Location L.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

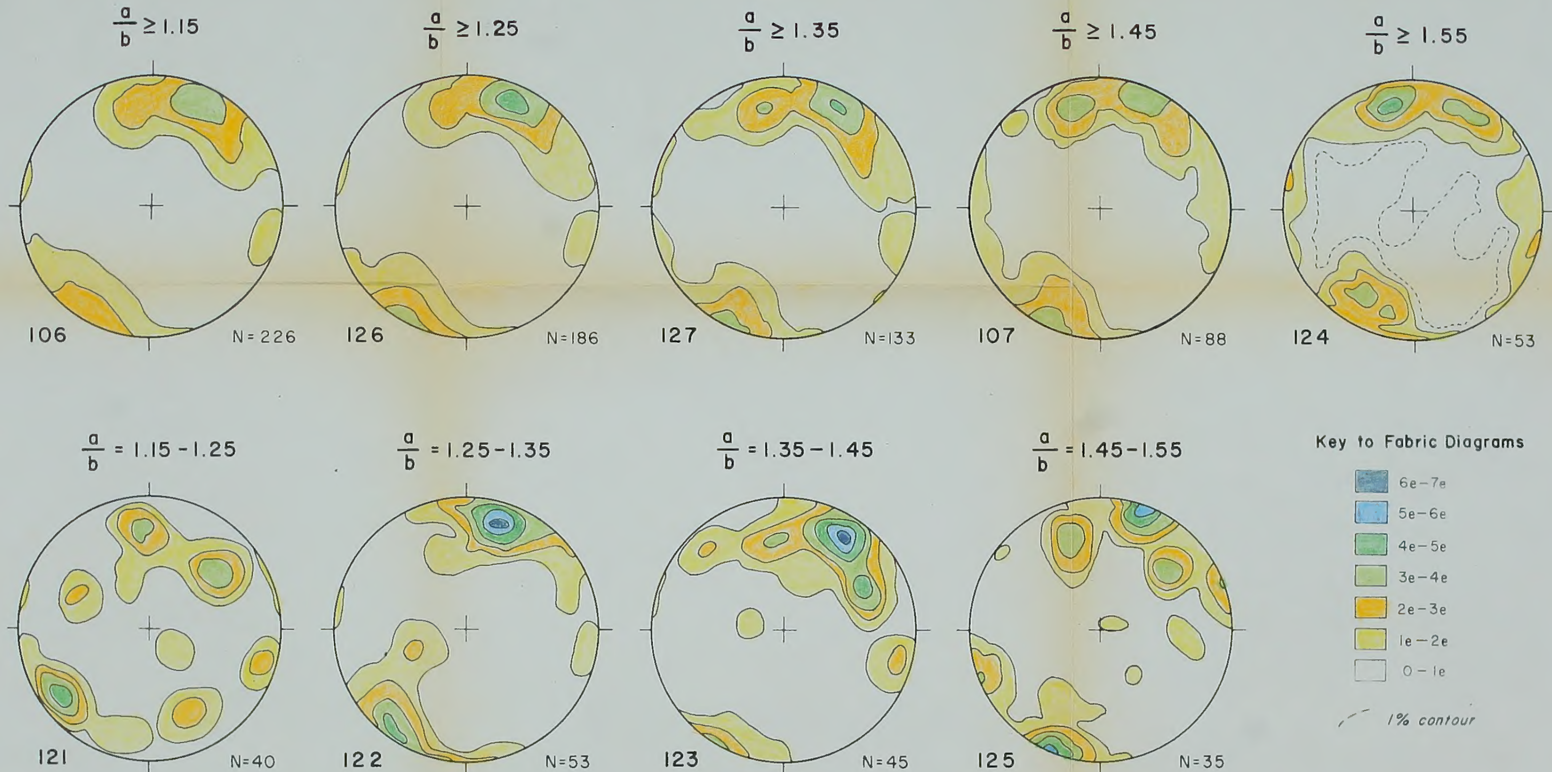
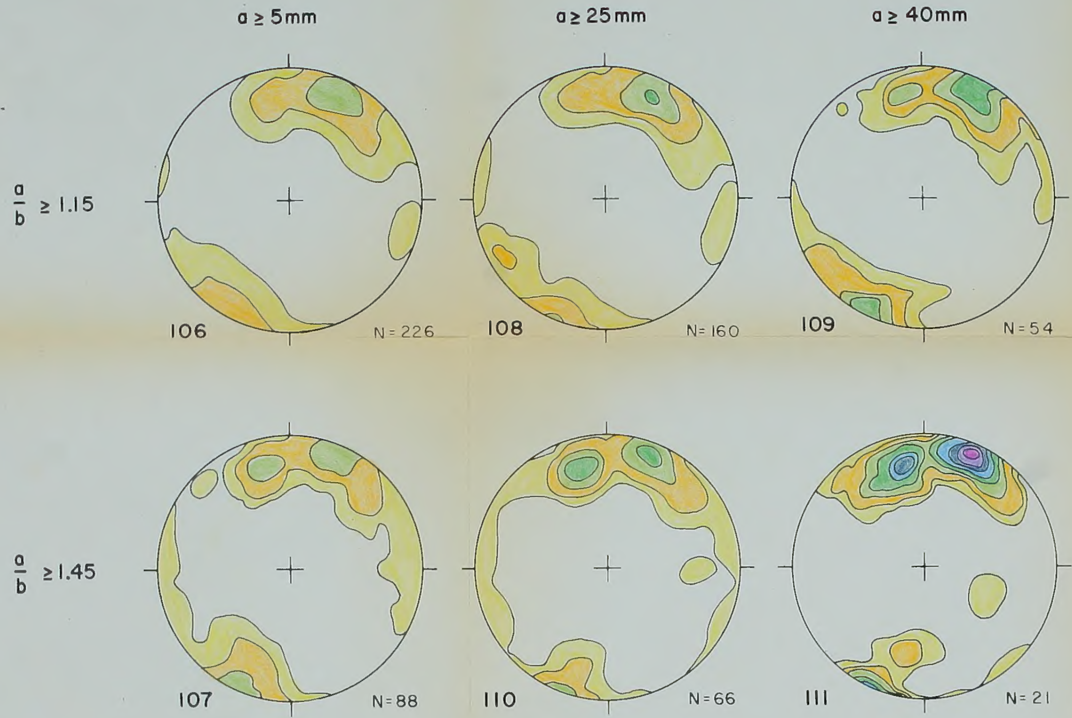
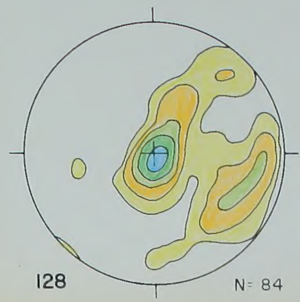


FIGURE 14. Effect of a:b ratio on a-axis orientation. Data from sites 34, 35 and 36 at Location F.

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.



A. Effect of a-axis length on a-axis orientation in two shape categories. Data from sites 34, 35 and 36 at Location F.



B. Poles of a:b planes from sites 34, 35 and 36 at Location F.

Key to Fabric Diagrams

9e - 10e	4e - 5e
8e - 9e	3e - 4e
7e - 8e	2e - 3e
6e - 7e	1e - 2e
5e - 6e	0 - 1e

FIGURE 15

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.

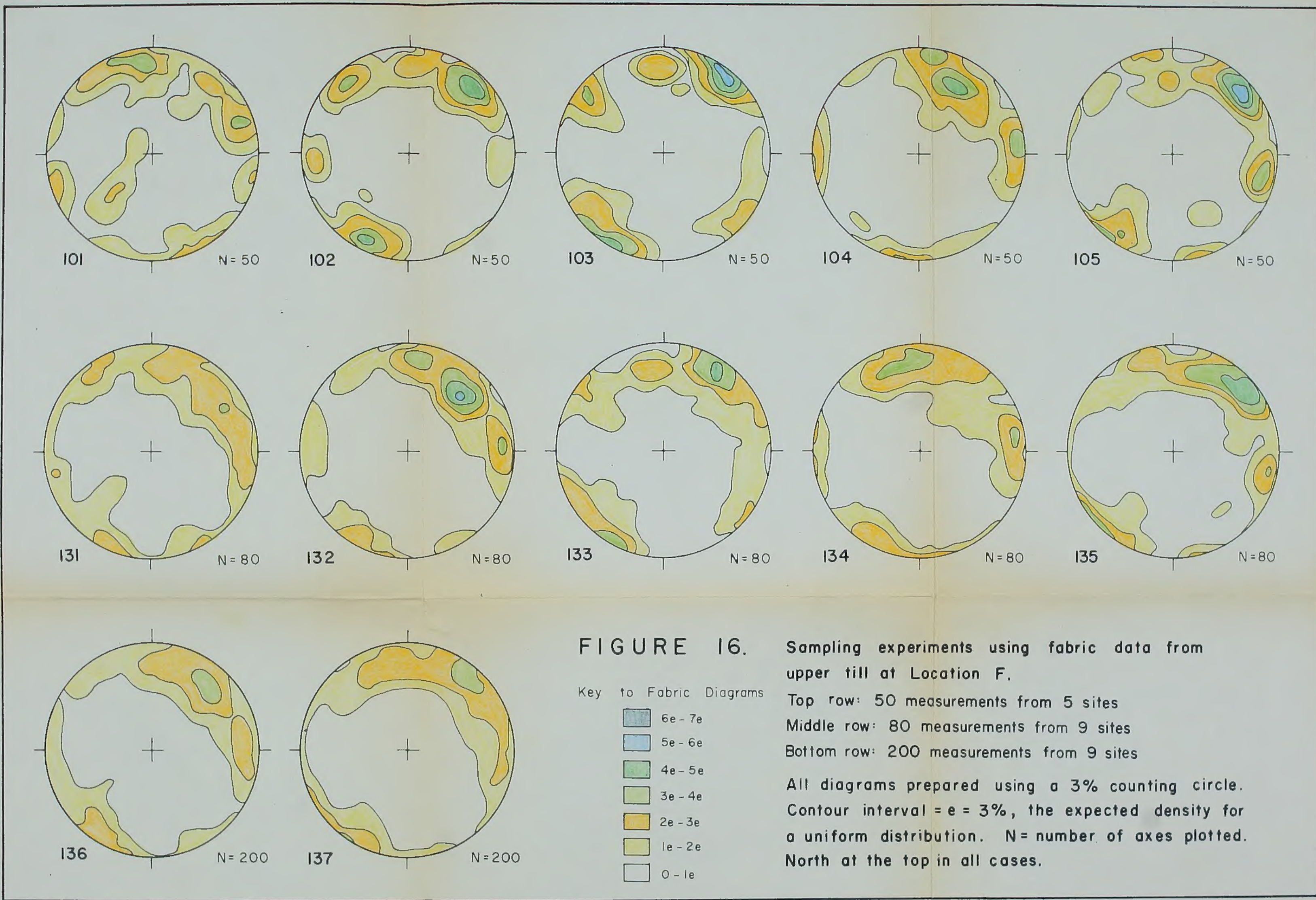


FIGURE 16. Sampling experiments using fabric data from upper till at Location F.

Top row: 50 measurements from 5 sites

Middle row: 80 measurements from 9 sites

Bottom row: 200 measurements from 9 sites

All diagrams prepared using a 3% counting circle. Contour interval = e = 3%, the expected density for a uniform distribution. N = number of axes plotted. North at the top in all cases.